On the Use of Splines for Wavelet Construction for Solving the Problem of Biomedical Signal Analysis Process Automation

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Automating the biomedical signal analysis process is a relevant objective that allows to:

- Apply a mobile device to control the physiological conditions of a human.
- Ensure continuous monitoring during remote treatment.
- Duly diagnose socially significant diseases and prevent serious consequences of their development.
Such systems can be implemented on different hardware components:

- Digital signal processors (DSP).
- FPGA.
- System-on-chip (SoC).
Wavelets

Wavelets are special class of functions with zero integral value capable of shift along the time axis and scaling.

Electroencephalogram

Electroencephalogram is a compound signal which can be detected on the surface of a human head and is the result of summation and filtration of the potentials of the groups of brain neurons.
Continuous wavelet transform

The continuous wavelet transform of the function $f(t)$ is as follows:

$$W(a, b) = \frac{1}{|a|^{1/2}} \int_{-\infty}^{\infty} f(t) \psi \left( \frac{t-b}{a} \right) dt,$$

where $a$ is the scaling coefficient value, $b$ is the shift along the time axis.
Wavelet synthesis procedure for continuous wavelet transform comprises the following steps:

A. Selection of signal fragment on the basis of which wavelet synthesis will be performed. This fragment will be called a sample.

B. Fragment valuation, derivation of a vector defined on the interval $[0,1]$.

C. The mathematical description of the sample in order to obtain the function the value of which can be calculated at any time, which is necessary for implementation of the algorithms based on the continuous wavelet transform on a different element basis with varying frequency of sampling of the analyzed signal.
Wavelet synthesis procedure for continuous wavelet transform

D. To perform the inverse continuous wavelet transform:

\[ f(t) = C^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(a, b) \psi \left( \frac{t - b}{a} \right) \frac{1}{a^{1/2}} \frac{dadb}{a^2} \]

it is necessary for the normalizing factor to be as follows:

\[ C = \int_{-\infty}^{\infty} |\psi|^2 |\omega|^{-1} d\omega < \infty \]

which is the condition for admissibility.

E. Modification of the mathematically described sample in order to obtain a function satisfying the admissibility condition.

F. Adding wavelet to the wavelet bank.
The use of splines in the synthesis of wavelets

Wavelet synthesis procedure for continuous wavelet transform comprises the following steps:

A. Selection of the sample.

Fragment of EEG with pathological activity

Sample
The use of splines in the synthesis of wavelets

B. Let us perform the sample valuation and obtain the vector defined on the interval \([0,1]\).

Sample after the procedure of normalizing
The use of splines in the synthesis of wavelets

C. Mathematical description of the sample.

By the cubic spline, we mean function $S$ which at each interval $[x_i, x_{i+1}]$ is a cubic polynomial of the type:

$$S(x) = a_{i,0} + a_{i,1}(x - x_i) + a_{i,2}(x - x_i)^2 + a_{i,3}(x - x_i)^3$$

$$x \in [x_i, x_{i+1}] \quad i = 0, ..., N - 1$$

Interpolation of the sample
The use of splines in the synthesis of wavelets

D. Let us test the function obtained during the interpolation for the conditions of admissibility.

E. Let us complete the modification of the function.

F. The resulting wavelet can be placed into a wavelet bank for storage on your hard disk.
Wavelet synthesis scheme

1. The sample signal
2. Sample valuation
3. Interpolation by cubic splines
4. Test the condition of admissibility
5. Modification of the sample
6. Putting the wavelet into a wavelet bank
Performance of the continuous wavelet transform using synthesized wavelets

- Analyzed signal and wavelet-spectrogram obtained by using Gaussian wavelet
- Analyzed signal and wavelet-spectrogram obtained by using synthesized wavelet
Conclusion

The main results obtained in this work are as follows:

1) Application of splines in the synthesis of wavelets for continuous wavelet transform allows to perform the analysis of wavelet mathematical model that provides localized mapping features on wavelet spectrogram and hence can be used to implement automatic biomedical signal analysis algorithms.

2) The advantages of this approach of the construction of wavelets are accuracy of the mathematical model with respect to the sample, as well as the possibility of synthesis based on wavelet smooth signals.

3) The disadvantage of the resulting model is the large number of parameters needed to describe it.
4) The resulting synthesized wavelets may also be used for inverse continuous wavelet transform, which has been tested in a practical way.

5) Implementation of all the algorithms which are based on the continuous wavelet transform requires considerable computational costs. In the analysis of multi-channel signals, it is necessary to use the element base that allows you to split computing threads. In case of implementing such systems in the form of mobile devices, we need to pay special attention to noise immunity and low power consumption.
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Thank you for your attention.

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