Impact of Node Placement on the Connectivity of Wireless Ad Hoc Networks

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***Abstract*—In the development of Internet of Things (IoT), one of the directions is the deployment of ubiquitous sensor networks (USN). Actually, these networks are the wireless Ad Hoc networks and their properties depend on the node placement and node behavior. One of the main characteristics is connectivity, which reflects the ability to provide services. Results of the USN connectivity research at various ways of placement of nodes and dependence of the connection to the main network settings are obtained.**

I. INTRODUCTION

Wireless self-organizing network (USN) may be defined by a plurality of nodes, each of which is in access zone of at least one node of the plurality, and each node has the ability to send data to a node - Gateway or any other network node [1].

Network properties depend on the characteristics of the components and their relative position. In general, a heterogeneity net all or part of the nodes may be mobile, have different performance characteristics and standards on the physical and channel levels. Network properties depend on many factors: the characteristics of the service zone, the distribution of nodes in space, their parameters, etc.

One of the most important factors, affecting the properties of the network, is its topology, i.e., the location of the nodes relative to one another in the service area. The network topology is largely determines the choice of technologies of physical and channel levels, self-organization protocols. In turn, the selection process of network nodes independent of its location destination node parameters, the method of installation.

If the nodes are "linked" to some objects of the maintained infrastructure (e.g., controlled objects), their location is determined, firstly, by the placement of these objects, as well as the manner of some accommodation units that perform ancillary functions. In case of such "binding", the location of nodescan be considered deterministic, where the coordinates of nodes and the distance between them are known.

In case of a network with fixed (stationary) nodes, depending on its purpose, the distribution of nodes in the service area may be performed in various ways. The following tasks may be solved: covering a region or regions in the service area of the zones of action of sensor devices that are part of the network nodes, providing network connectivity and reliability.

Generally, placement of nodes may be considered as random. In real network with deterministic placement of nodes, it is impossible to provide the absolute accuracy of their installation [2], thus it does not contradict the assumption of the random nature of their placement.

In case of binding nodes (e.g., individual devices similar to mobile phones) to users, the distribution of nodes is defined by the distribution of users (subscribers). In case of binding nodes to things surrounding a person, the distribution of nodes in space is defined by the distribution of these things.

The topology of wireless Ad Hoc network greatly affects the main indicators of its operation such as availability and message delivery latency [3], [4].

As a rule, the concept of availability is considered as possibility of providing a service. For wireless Ad Hoc networks, it is tightly coupled with the concept of connectivity.

Similar to graph connectivity [5], network is connected if there is at least one route for data delivery between any couple of nodes. The probability of connectivity of a network defining a quantitative share of possible connections and being the major characteristic [6].

Message delivery latency is also one of the main indicators of the network performance and depends on wireless communication technology and the number of transits in the route [7].

Wide variety of realization and scopes of the wireless Ad Hoc networks results in need of the analysis of their properties and definitions of methods of key parameters assessment [8], [9].

In various conditions, features of their topology characterized networks. For studying the topology of USN properties appropriate to consider different ways of placement of nodes (and their parameters), defining a set of characteristics of these processes are placed.

Thus, the research objective of this paper is to study the connectivity of a network as a function of the distribution of nodes and the impact of the distribution of nodes and their

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parameters on the efficiency of the network with the set requirements to connectivity.

The goal of investigation is to determine the dependence of the network connectivity of the distribution of its sensor nodes.

II. Probability of Network Connectivity

The wireless Ad Hoc network consists of number of nodes n, each may be connected or not to the neighbornodes. In the latter case, data delivery isn't possible for this node. The arrangement of nodes depends on a specific purpose of the network and it is probable that it gets out taking into account ensuring connectivity. However, during operation nodes can deny or change their position, for example, in the case of network with mobile nodes. Therefore, it can be assumed that the distribution nodes randomly, therefore the links between them are also random. We make the assumption that the number of nodes is always equal to n. In this case, the network can be described as random graph G (n, p), where p – likelihood of a connection between nodes.

Well-known Erdos-Renyi model [5], [10] allows describing probability of connectivity of the random count $G = \{V_n, E\}$.

The model of network is set by the count containing nodes $V_n = \{1,...,\mathrm{n}\}$, existence of an edge between tops of I and j in the column is defined by probability p_{ij}, E – random set of edges in the column.

Then, the probabilistic space is defined as

$$G\left(n, p_{ij}\right) = \left(\Omega_n, F_n, P_{n,p_{ij}}\right), \tag{1}$$

where

$$\Omega_n = \{G = G(V_n, E)\},\quad F_n = 2^{\Omega_n},\quad \mathrm{P}_{n,p_{ij}}(G) = \prod_{(ij)\in E} p_{ij} \prod_{(ij)\notin E} (1 - p_{ij})$$

If to fix some count $G = \{H_n, p\}, H_n = \{V_n, E_n\}$,

where

$$p_{ij} = \begin{cases} p, & (\mathrm{i,j}) \in \mathrm{E}_n \\ 0, & (\mathrm{i,j}) \notin \mathrm{E}_n \end{cases},$$

$$P_{n,p_{ij}}(\mathrm{G}) = \mathrm{p}^{|E|}(1-\mathrm{p})^{|E_n|-|E|}, \tag{2}$$

where $|E|$ - the expected number of edges in the graph,

than for the random counts, described by this model, known the theorem [10] which says: if known that $p = \frac{c \ln n}{n}$, where c – constant, at c<1 graph is never coherent, and at c<1, graph is practically always coherent.

Actually, expression $p = \frac{\ln n}{n}$ defines a threshold value of probability of an edge at which excess the network is coherent with probability more than 0.5.

For large enough n and c, the probability of connectivity of the count can be approximately estimated [10] as

$$P_{n,p}(\mathrm{G}) = 1 - \frac{1}{n} \tag{3}$$

III. Probability of Network Connectivity in Case of Poisson Field

Consider two options of network organization: network with nodes forming Poisson field [11], in the region bounded by a square with sides of 200 m and network with nodes forming Gaussian field.

In terms of Gaussian field we understand the network model, where coordinates of nodes are random, independent and distributed according to the two-dimensional normal distribution (Fig.1).

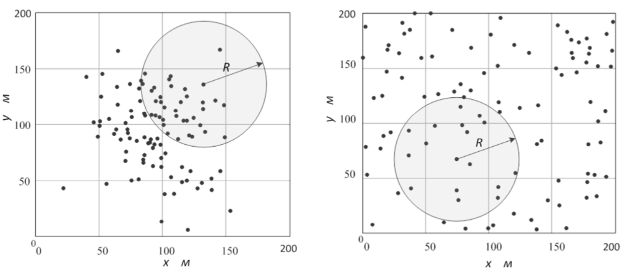

Fig.1. Node placements examples in case of Poisson and Gaussian fields

In both cases, the node of network has limited area of communication which is described by circle with the center in a point of placement of node and radius R.

Probability of existence of the edge between tops of graph p, representing the considered network, will be defined as

$$p = \frac{E(R)}{n}, \tag{4}$$

where $E(R)$ - expected value of number of adjacent nodes, i.e., the number of nodes from the reprting node at a distance less than R.

$$E(R) = \rho * s_R = \rho \pi R^2, \tag{5}$$

where $\rho = \frac{n}{S}$ (nodes/m^2) density of nodes in network,

$s_R = \pi R^2$ (m^2) - square of a circle with radius R.

Fig. 2 shows the results of simulation modeling network whis 100 nodes in a region bounded by a square with a side of 200 m.

As a result of simulation obtained estimates of the probability of connectivity. Dependence p(R) (4) is also shown on Fig. 2.

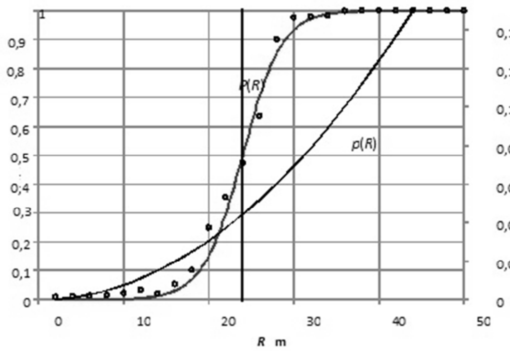

Fig.2. Network connectivity as a function of communication range of Poisson field node

As seen from the results, the probability of connectivity of a network is equal 0.5 at p close to theoretical value $p = \frac{\ln n}{n} = \frac{\ln 100}{100} \approx 0.046 \cdot$

According to the above-mentioned theorem, this value is the probability of a boundary that defines the point of phase transition the network from a disconnected state to a connected and vice versa.

Received result shows that application of the Erdos-Renyi model for the network determined by a Poisson field of points gives quite exact results and allows to estimate, in this case, the necessary density of nodes or radius of communication of a hub R for ensuring necessary connectivity of network.

Note that the width of the phase transition (in this example, about 20 m radius site) characterizes the resistance network in terms of ensuring connectivity changing range communication, for example, the reception conditions due to noise or the environment.

IV. NETWORK CONNECTIVITY PROBABILITY IN CASE OF GAUSSIAN FIELD

As it was noted above, in many practical tasks distribution of nodes of network differs from a Poisson field, generally, it can be multimodal with areas of high and small density of nodes.

To study the properties of the field, other than the Poisson, consider a network formed by units stationed in the area under the normal law of service, i.e. forming a Gaussian field points in the plane. Generally, this model can be used in non-uniform distribution of nodes in the considered network service area.

Assume, that density of nodes in each point of surface is random and set by casual, independent coordinates x and y. Then the probability density function will be determined by the joint distribution function of the random x and y. For a normal distribution with a scattering center at (μx, μy) and a circular dispersion (equality of dispersions on x and y) density distribution is equal

$$\rho(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{(x-\mu_x)^2+(y-\mu_y)^2}{2\sigma^2}}, \quad (6)$$

where σ - standart deviation.

Assume, that the probability of hit in a zone of service is equal to1. The normal law of distribution of probability is boundless on values x and y. Strictly speaking, in these conditions the law of distribution can't be normal. However, with a sufficient accuracy it can be described "the distribution truncated normal" [12]

$$\rho(x,y) = K(\sigma,\mathrm{R}) \cdot \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}, \quad (7)$$

where

$$K(\sigma,R) = \frac{1}{\iint\limits_{S_R} \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}}, \quad (8)$$

where S_R - area of service.

Choosing rather small value σ, it is possible to achieve that $\iint\limits_{S_R} \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \succ \varepsilon$, where ε-number, rather close to 1.

Then, with a sufficient accuracy for the description of nodes density distribution expression (6) can be used.

Results of simulation modeling of network from 100 nodes with the radius of communication range of node - 50 m are given in Fig 3.

We used the normal law of nodes distribution with the center of dispersion in the center of a square, i.e. $\mu_x = \mu_y = 100\ m$, and standard deviations equal to a mean square deviation

$$\sigma_x = \sigma_y = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{200^2}{12}} \approx 57,74\ m$$

in the area limited by square with the side of 200 m.

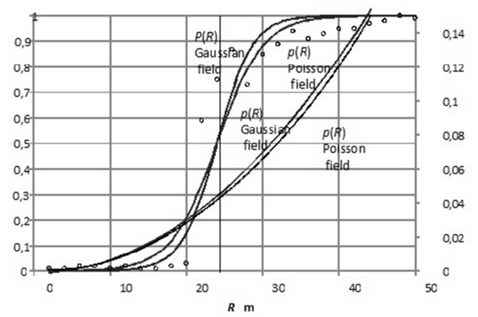

Fig. 3.Network connectivity as a function of communication range of node for Gaussian and Poisson fields

From the given results of simulation modeling of Gaussian field it is visible that phase transition is close to phase transition for a Poisson field. The value of probability p (R), given on the same drawing also received as a result of simulation modeling shows justice expression (3) in relation to Gaussian field. However, in this case for calculation p expression (4) does not apply, because the expected value of the number of nodes in the region, bounded by a circle, depends not only on its radius R, but also on the choice in the position of its center (x_0, y_0)

$$E(R, x_0, y_o) = n \iint\limits_{C(R, x_0, y_0)} f(x, y)\,dxdy, \qquad (9)$$

where $C(R, x_0, y_0)$ - integration area, for example, the considered circle with the center in a point (x_0, y_0) and radius R, $f(x, y)$ - two-dimensional normal distribution (6).

Rank p is a certain difficulty, in addition, an analytical solution (9) is possible only in special cases. However, the results will allow suggest that the probability p is determined by the dispersion of the network nodes. The dependence of the probability p of the variance for a Gaussian distribution obtained by simulation is shown in Fig. 4.

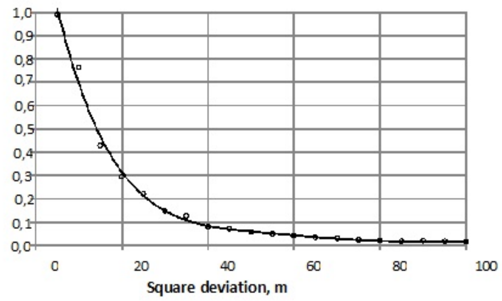

Fig. 4.Probability p as a function of the dispersion

Simulations have shown that a network connectivity, formed as Poisson and Gaussian fields distribution determined by the dispersion of nodes in the territory.

It should be noted, that in this case the network was considered in general. In such conditions it is obvious that the same-dimensional distribution (Poisson field) conditions for the connection of network nodes, located in different coordinates of the service area, equal (equal-probability).

For a network whose nodes are distributed in accordance with the normal distribution (Gaussian field), the conditions are not equal, since the density of nodes and the probability of connectedness for them depend on the coordinates, i.e. on the distance from the scattering point.

For network with nodes distributed in compliance with normal distribution (Gaussian field) these conditions aren't equivalent since density of nodes and probability of connectivity for them depends on their coordinates, namely on removal from a dispersion point.

For simplification of connectivity analysis of nodes we will make the following assumptions.

We will draw around the scattering center some circles of equal density and various radius (Fig. 5), multiple R (kR). In our example three circles with radiuses of R, 2R and 3R. We will assume that density of nodes in first circle (radius R), and other rings (2R and 3R) is constant.

For normal distribution (in the special case in circle of equal density with radius r), the probability of hitting the circle is

$$P_r = P\big((x, y) \in B_k\big) = 1 - e^{-\frac{r^2}{2\sigma^2}}, \qquad (10)$$

Respectively the probability of getting into the ring k is equal

$$P_{C_k} = e^{\frac{((k-1)R)^2}{2\sigma^2}} - e^{\frac{(kR)^2}{2\sigma^2}}, \; k = 1, 2... \qquad (11)$$

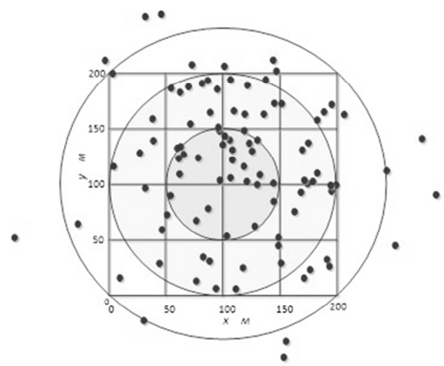

Fig.5. Nodes Distribution in the service area

In the conditions of assumption about the equal density of nodes in borders of rings, the probability in borders of rings will be defined as

$$p_k = \frac{\rho_{Ck} \pi R^2}{n}, \qquad (12)$$

where density of nodes in k ring - $\rho_{Ck} = \frac{kP_{Ck}}{S_{Ck}}$, (13)

square of k-ring - $S_{Ck} = \pi\left[(kR)^2 - ((k-1)R)^2\right]$ (14)

Dependence of probability on the number of ring, i.e. on removal on the center of dispersion is given in Fig. 6.

As might be expected, dependence shows that the probability of connectivity decreases as the distance from the scattering center.

In addition, for the rings, starting from the third (the distance from the scattering center is more than 150 m), the probability p3 is less than the threshold value 0.046, which means that the probability of a connectedness in this and further away is less than 0.5.

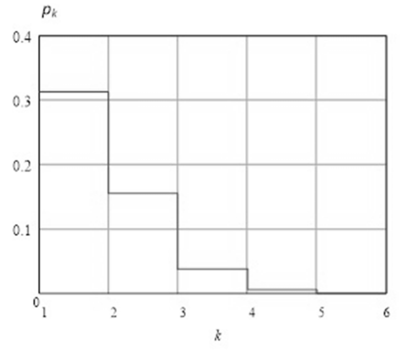

Fig.6. Dependence p_k on the distance from the center of dispersion

Thus, using (11) and (12) we can estimate such nodes distribution parameters as nodes number n, and the standard deviation σ (for example, for a given distance between the scattering center), in which all parts of the network are provided the requirements for probability of connection.

V. CONCLUSIONS

These studies produced the following results:

Random nature of nodes distribution of wireless Ad Hoc network on territories and random nature of properties of radio channels between nodes allow to use model of the random graph as network model.

By means of simulation modeling it is shown that the factor defining connectivity of network is dispersion of nodes distribution across the territory.

Application of Erdos-Renyi model allows to define connectivity of network using such parameters as the number of nodes, radius of communication of node, dispersion of their distribution across the territory.

Results of modeling for Poisson and Gaussian fields showed independence of probability of network connectivity in general from distribution type.

The results of the analysis of Gaussian distribution shows the dependence of connectivity in the field of network from its distance from the center of dispersion. Received an expression that allows to evaluate the connectivity of network in a ring of equal density proceeding from such parameters as: number of nodes, radius of communication of node and dispersion of distribution of nodes across the territory.

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