On the Use of Splines for Wavelet Construction for Solving the Problem of Biomedical Signal Analysis Process Automation

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***Abstract*—The article describes the procedure of wavelet synthesis for continuous wavelet transform and opportunities to apply cubic splines for this. As the analyzed signal one of biomedical signals is given, the signal is EEG. All phases of the construction of a new wavelet based on a signal fragment are described. The results are obtained by applying wavelets derived in the process of direct and inverse continuous wavelet transform. At the end we provide recommendations on selecting a digital signal processor that can be used when implementing automatic biomedical signal analysis algorithms.**

I. Introduction

Automating the biomedical signal analysis process is a relevant objective that allows to:

- Apply a mobile device to control the physiological conditions of a human.
- Ensure continuous monitoring during remote treatment.
- Duly diagnose socially significant diseases and prevent serious consequences of their development.

Such systems can be implemented on different hardware components: digital signal processors (DSP), programmable logic integrated circuits (FPGA), system-on-chip (SoC), and in some cases on general purpose processors.

It is obvious that such devices should relate to digital signal processing systems. As is known, the design of such systems includes the following steps [1]:

1) Development of mathematical apparatus.

2) System modeling in mathematics software package, for example in MATLAB.

3) Implementation.

Thus, the key factor in the development of such systems is the selection or development of the mathematical apparatus that allows to get an algorithm of effective solution for all the specific tasks to achieve the projected accuracy of the designed diagnostic equipment.

The paper suggests using the mathematical apparatus of wavelets in the automation of the process of analyzing biomedical signals.

Wavelets are special class of functions with zero integral value capable of shift along the time axis and scaling [2, 3].

Wavelets are widely applied in the analysis of various signals, including biomedical, such as electrocardiogram, electroencephalogram, electrogastrogram and others [4].

Let us consider electroencephalogram as an example of a biomedical signal.

Electroencephalogram is a compound signal which can be detected on the surface of a human head and is the result of summation and filtration of the potentials of the groups of brain neurons [5].

Electroencephalogram is recorded simultaneously on several channels, according to electrode leads, located on different parts of the patient's head [6, 7].

In the analysis of electroencephalogram it is important to identify its basic rhythms as well as graphoelements reflecting the physiological condition of a human and artifacts (the phenomena which are not directly related to the activity of the human brain).

In the process of detecting such graphoelements (referred to as features) continuous wavelet transform can be used.

The continuous wavelet transform of the function $f(t)$ is as follows [8–13]:

$$W(a,b) = \frac{1}{|a|^{1/2}} \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-b}{a}\right)dt,$$

where a is the scaling coefficient value, b is the shift along the time axis.

The result of the continuous wavelet transform is wavelet spectrogram.

The localization of features on wavelet spectrogram is greatly influenced by the choice of a wavelet.

Studies have shown that the traditional families of wavelets do not provide the localization of features on wavelet spectrogram, which is necessary to automate the process of the analysis.

It is necessary to carry out the construction of new wavelets similar to the analyzed signal in their characteristics.

II. Wavelet Synthesis Procedure for Continuous Wavelet Transform

Wavelet synthesis procedure for continuous wavelet transform comprises the following steps:

A. Selection of signal fragment on the basis of which wavelet synthesis will be performed. This fragment will be called a sample.

B. Fragment valuation, derivation of a vector defined on the interval [0,1].

C. The mathematical description of the sample in order to obtain the function the value of which can be calculated at any time, which is necessary for implementation of the algorithms based on the continuous wavelet transform on a different element basis with varying frequency of sampling of the analyzed signal.

D. To perform the inverse continuous wavelet transform:

$$f(t) = C^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(a,b) \psi\left(\frac{t-b}{a}\right) \frac{1}{a^{1/2}} \frac{dadb}{a^2},$$

it is necessary for the normalizing factor to be as follows:

$$C = \int_{-\infty}^{\infty} |\hat{\psi}|^2 |\omega|^{-1} d\omega < \infty,$$

which is the condition for admissibility [14].

To verify the fact that the function is a wavelet it is enough to have its integral value which is equal to zero.

E. Modification of the mathematically described sample in order to obtain a function satisfying the admissibility condition.

F. Adding wavelet to the wavelet bank.

III. The Use of Splines in the Synthesis of Wavelets

For the mathematical description of the sample various methods of approximating functions can be used. One of the objectives of this paper is to consider the possibility to use the interpolation by cubic splines in the wavelet synthesis procedure for continuous wavelet transform. As is well known [15], [16], cubic splines allow to present signal fragments even of a very complicated shape with high accuracy and sufficient smoothness.

In this case, the synthesis procedure will include the following steps:

A. Selection of the sample.

At this stage, you need to select a fragment of the signal to be used in the future as the basis for the synthesis of a wavelet. If you select this fragment, you need to have an idea about the characteristics of the features that need to be identified in the signal – the shape, the frequency and the amplitude.

Electroencephalogram has a significant number of features that can be divided into two types:

1) Fragments of the electroencephalogram which can be attributed to pathological activity.

2) Fragments of the EEG that are not associated with the activity of the human brain, and the reason for their appearance is caused by the action of external factors, violations during the electroencephalographic study or involuntary muscle activity of the patient.

We can distinguish the following types of artifacts [5]:

- Eye artifacts. This type of artifact occurs in the process of EEG recordings when the patient produces eye movements. In this case there appear graphoelements of typical shape, amplitude and frequency (Fig. 1). Eye artifacts are pronounced in the frontal leads and damp out as we move towards the back of the head.

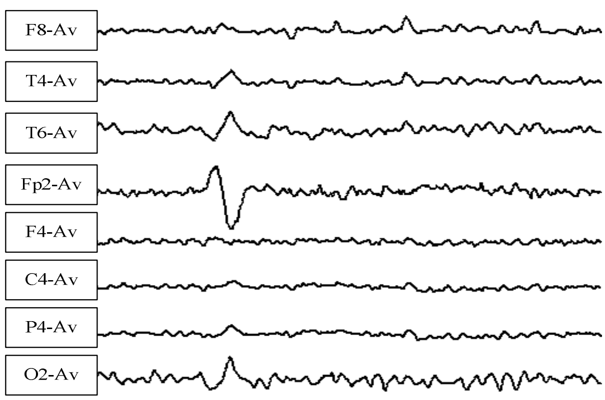

Fig. 1. EEG fragment with eye artifacts (Fp2-Av)

- Muscle artifacts related to the muscular activity of the patient in the process of electroencephalographic study. There appear high-frequency oscillations of considerable duration (Fig. 2).

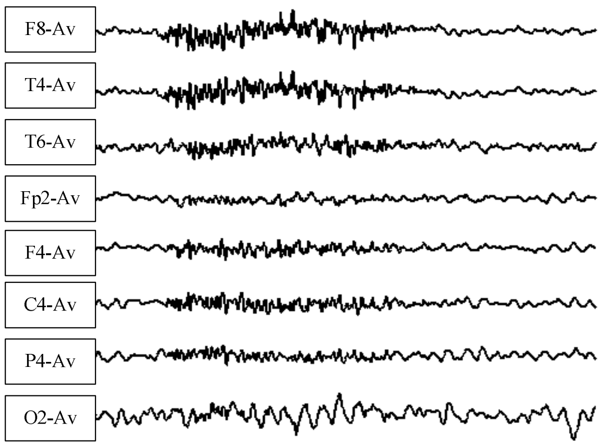

Fig. 2. EEG fragment with muscle artifacts

- Artifacts related to the displacement of the electrode during the electroencephalographic study.
- Artifacts associated with the location of the electrode in close proximity to the vessel, which leads to specific variations resembling an electrocardiogram in shape.
- Artifacts associated with proximity of electricity mains and photostimulator. They are displayed as typical oscillations with frequency of 50 Hz (60 Hz) and the frequency of stimulator outbreaks.

All kinds of these features need to be identified. In the case of pathological activity, it is necessary for diagnosis. In the case of artifacts, it is important to exclude them from the analysis of the electroencephalogram to avoid false interpretations of the physiological state of the human brain.

Artifacts associated with the current superposition of commercial frequency can be suppressed by using appropriate filters. Other artifacts should be excluded from the analysis automatically by being identified in the signal. As the device that performs the analysis may have a mobile implementation and may be used in conditions that are not suitable for electroencephalographic study, many ways of dealing with artifacts taken for stationary electroencephalographs are not available. For example, the exclusion of interference photostimulator by removing it at the distance required from the rest of the equipment is not always possible in a mobile electroencephalograph implementation.

Let us consider one of the fragments of the pathological activity as an example (Fig. 3).

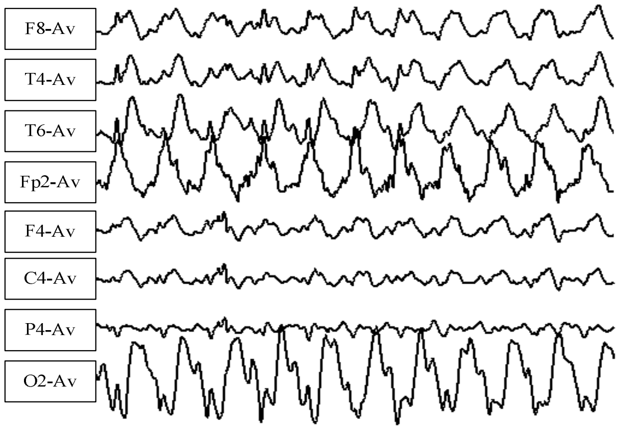

Fig. 3. Fragment of EEG with pathological activity

In identifying such features it is appropriate to apply wavelet that allows to localize on the wavelet spectrogram its every single graphoelement. Therefore, as a sample (Fig. 4) in the synthesis of the wavelet a short part of the record common for this type of pathological activity can be selected. The sampling frequency shown in Fig. 4 of the electroencephalographic signal is 256 Hz, so the sample is only represented by 128 references.

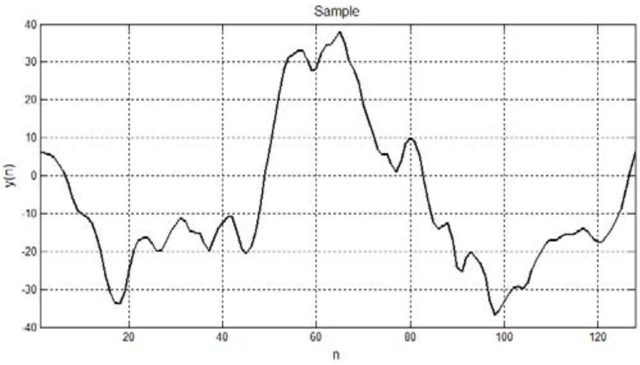

Fig. 4. Sample

B. Let us perform the sample valuation (Fig. 5) and obtain the vector defined on the interval [0,1].

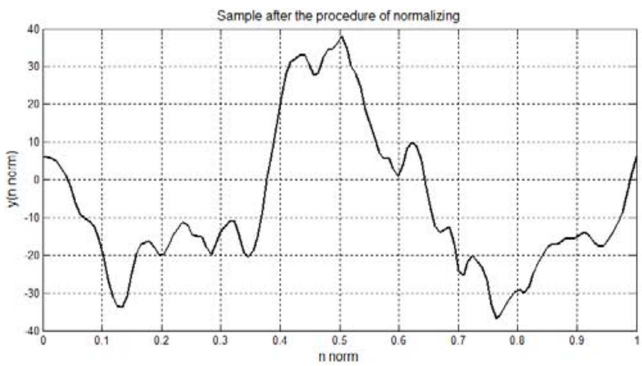

Fig. 5. Sample after the procedure of normalizing

C. Mathematical description of the sample.

The author suggests that in the mathematical description interpolation by cubic splines should be used.

By the cubic spline [15], we mean function S which at each interval $[x_i, x_{i+1}]$ is a cubic polynomial of the type:

$$S(x) = a_{i,0} + a_{i,1}(x - x_i) + a_{i,2}(x - x_i)^2 + a_{i,3}(x - x_i)^3 ,$$
$$x \in [x_i, x_{i+1}] , i = 0,..., N - 1 ,$$

and has a smooth blending of adjacent polynomials.

Let us perform interpolation of the sample by cubic splines (Fig. 6).

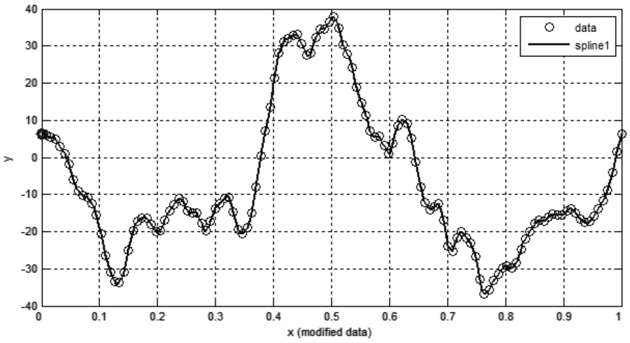

Fig. 6 Interpolation of the sample

High accuracy of approximation of the resulting function to the sample is obvious. This is due to the fact that the interpolation was carried out on a regular grid - the nodes coincide with the values of the discrete signal. Fig. 6 brings us to the conclusion about the smoothness of the resulting function. All of this can be attributed to the advantages of this method of mathematical description of the function in the synthesis of wavelets for continuous wavelet transform. As for disadvantages of this approach, there are a large number of parameters of the resulting mathematical model and, as a consequence, its high complexity. However, this disadvantage is not critical. Modern personal computers allow one to work with mathematical models with such number of parameters and, if necessary, to save them on hard drive.

D. Let us test the function obtained during the interpolation for the conditions of admissibility. For the function shown in Fig. 6, these conditions are not met.

E. Let us complete the modification of the function. In order to achieve zero integral value, the function may be shifted along the vertical axis. The author has written a script-file for MATLAB, which enables automatic modification of a function in order to achieve its zero integral value.

F. The resulting wavelet (Fig. 7) can be placed into a wavelet bank for storage on your hard disk. If necessary, the wavelet function can be loaded, the vectors of discrete values of the wavelet with the used element base and the sampling frequency of the analyzed signal can be calculated.

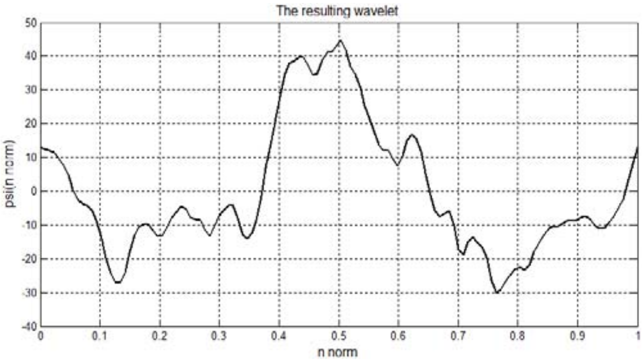

Fig. 7. The resulting wavelet

Thus the whole procedure of the synthesis of wavelet for continuous wavelet transform with the use of splines is reduced to the scheme shown in Fig. 8, where the dotted arrow shows an iterative procedure of the modification of the function.

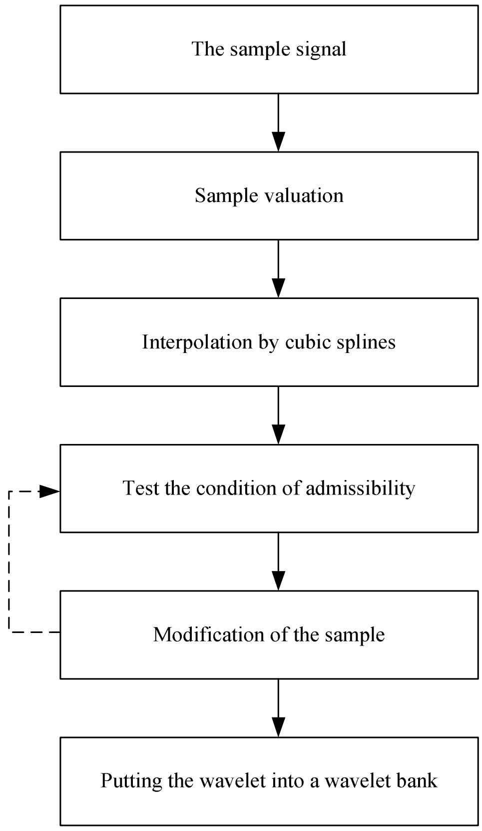

Fig. 8. Wavelet synthesis scheme

IV. Performance of the Continuous Wavelet Transform Using Synthesized Wavelets

The result of the continuous wavelet transform can be a wavelet spectrogram. Localized features mapped on wavelet spectrogram make it suitable for automated analysis because each feature finds its reflection in a light area, which corresponds to specific values of the wavelet coefficients at a given scale.

As a comparison, let us perform a continuous wavelet transform of the same signal fragment with pathological activity. In one case, we choose the Gaussian wavelet, and in another - the wavelet synthesized on the basis of a sample from the same type of characteristics, but taken from another electroencephalogram.

As can be seen from Fig. 9 and Fig. 10, the use of synthesized wavelet has made it possible to obtain the localized display of the features on the wavelet spectrogram.

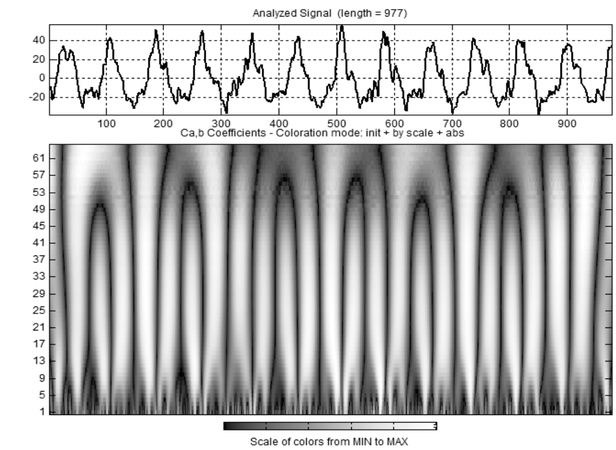

Fig. 9. Analyzed signal and wavelet-spectrogram obtained by using Gaussian wavelet

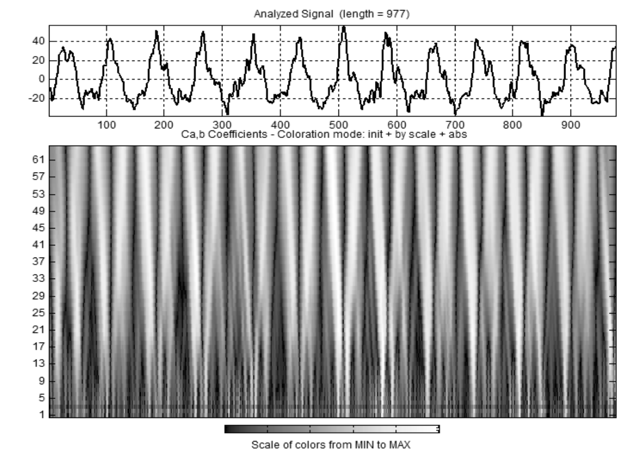

Fig. 10 Analyzed signal and wavelet-spectrogram obtained by using synthesized wavelet

V. The Inverse Continuous Wavelet Transform

The verification of the admissibility conditions or sufficient condition of admissibility is the theoretical basis for the possibility of the reverse procedure of continuous wavelet transform. In this case, the resulting function can be called a wavelet. The results obtained in this paper wavelets satisfy these requirements. In addition, to test the possibility of the inverse continuous wavelet transform a script file for MATLAB has been developed, this allows to carry out this conversion on the basis of wavelet coefficients obtained for a certain value scale.

Thus, it was shown in a practical way, that the resulting functions are wavelets synthesized for continuous wavelet transform.

VI. Implementation on the Basis of Digital Signal Processor

Let us consider one of the most common variants of the implementation of digital signal processing system which is the implementation of digital signal processor paying attention to the requirements for this element base.

With the implementation of the continuous wavelet transform on the digital signal processor it is advisable to keep the designed wavelet in the form of two vectors - the reference vector and the vector that sets the value of the wavelet at certain moments of the discrete normalized time. For example, the wavelet can be given by the vector of wavelet function values (psi) with the number of points N = 256. In this case, the reference vector will have the same number of points N = 256 and the values in the range [0, 1].

It is important for the digital signal processor to have:

- High performance. Most of the algorithms, which are based on the continuous wavelet transform, require substantial computational cost, so only the processor with high performance can provide the system speed approximation to the speed of the real time.
- Large internal memory, enough to store the information used for continuous wavelet transform (or a set of wavelets), as well as to organize the process of loading, unloading and storing other necessary for computations information. Using external memory located on the target board, as is known, can significantly reduce the computational speed.
- The analysis of multichannel signals which also include electroencephalogram it is possible to apply multi DSP. It is also possible to use digital signal processors for sharing computer flows on each core.

- Low power consumption, which is important for the creation of truly mobile systems with a significant battery performance based on internal battery.
- Good protection against transients [17]. It is obvious that the interference from impulse noise may cause a malfunction of the system. The source of impulse noise can include different systems, those which are faced every day in the big city.
- The small size, which affects the size of the final product and the ease of porterage and use. For most modern digital signal processors, this requirement is fulfilled.
- Low Cost.

In order to relax the requirements for the process and thus reduce the operation costs, it is necessary to take measures to optimize the system algorithm and program code. It is possible to use Assembler language or C-language with inline assembly.

VII. Conclusion

The main results obtained in this work are as follows:

1) Application of splines in the synthesis of wavelets for continuous wavelet transform allows to perform the analysis of wavelet mathematical model that provides localized mapping features on wavelet spectrogram and hence can be used to implement automatic biomedical signal analysis algorithms.
2) The advantages of this approach of the construction of wavelets are accuracy of the mathematical model with respect to the sample, as well as the possibility of synthesis based on wavelet smooth signals.
3) The disadvantage of the resulting model is the large number of parameters needed to describe it.
4) The resulting synthesized wavelets may also be used for inverse continuous wavelet transform, which has been tested in a practical way.
5) Implementation of all the algorithms which are based on the continuous wavelet transform requires considerable computational costs. In the analysis of multi-channel signals, it is necessary to use the element base that allows you to split computing threads. In case of implementing such systems in the form of mobile devices, we need to pay special attention to noise immunity and low power consumption.

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References

[1] A.I. Solonina, *Digital Signal Processing. Simulation in Simulink.* St. Petersburg: BHV-Petersburg, 2012.
[2] *A.I. Solonina, S.M. Arbuzov, Digital Signal Processing. Modeling in MATLAB.* St. Petersburg: BHV-Petersburg, 2008.
[3] S.M. Arbuzov, A.B. Stepanov, *Application of wavelet analysis in electroencephalography.* St. Petersburg: Link, 2009.
[4] N.K. Smolentsev, *Fundamentals of the theory of wavelets. Wavelets in MATLAB.* Moscow: DMK Press, 2014.
[5] L.R. Zenkov, M.A. Ronkin, *Functional diagnosis of neurological diseases: a guide for doctors.* Moscow: MEDpress-inform, 2011.
[6] Y.D. Kropotov, *Quantitative EEG, cognitive evoked potentials of the human brain and neurotherapy.* Donetsk: Publisher Zaslavsky A.Y., 2010.
[7] V.V. Gnezditsky, *The inverse objective of EEG and Clinical electroencephalography (mapping and localization of sources of electrical activity in the brain).* Moscow: MEDpress-inform, 2004.
[8] N.K. Smolentsev, *Introduction to the theory of wavelets.* Moscow-Izhevsk: "Regular and Chaotic Dynamics", 2010.
[9] V.V. Vityazev, *Wavelet analysis of time series.* St. Petersburg: Publishing St. Petersburg State University, 2001.
[10] H.-G. Stark, *Wavelets and Signal Processing*. Berlin: Springer, 2005.
[11] C. Blatter, *Wavelets - Eine Einfuhrung*. A.K. Peters, Ltd., 1998.
[12] I. Daubechies, *Ten Lectures on Wavelets*. SIAM, 1992.
[13] C.K. Chui, *An Introduction to Wavelets*. Academic Press, 1992.
[14] M. Misiti, Y. Misiti, G. Oppenheim, J.-M. Poggi, *Wavelets and their Applications.* ISTE Ltd., 2007.
[15] B.I. Kvasov, *Methods of isogeothermal approximation by splines.* Moscow: FIZMALIT, 2006.
[16] V.A. Srochko, *Numerical methods. Lectures.* St. Petersburg: Publishing "Lan", 2010.
[17] A.I. Solonina, D.A. Ulakhovich, L.A. Yakovlev, *Algorithms and digital signal processors*. St. Petersburg: BHV-Petersburg, 2001.