Calculation and Optimization of Industrial Robots Motion

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Abstract—**This paper considers a problem of improving automated industrial manufacturing systems with a help of low-costs robotics manipulators with a programmed control. For widespread use at various industrial enterprises of robots-manipulators with programmed control without expensive sensors and elements of artificial intelligence, we use methods of determining the spatial and kinematic characteristics of the working body of the manipulator. The method of determining the kinematic characteristics is based on the matrix method in the kinematics of robots and the second-order Lagrange matrix equations in dynamics. The method allows to calculate the optimal modes of movement of the manipulator, to increase the speed of operations on the production line. The presented approach makes it possible to optimize the number of multipurpose manipulators with programmed control for various technological operations, and also allows to increase the productivity of a robotic production line.**

I. Introduction

Industrial robots are used in automated production systems. Automation of production with a help of robots allows to improve the quality of products, increase the accuracy of technological operations, increase labor productivity and eliminate exposure of harmful factors for workers.

In the production process, industrial robots perform the following main technological operations: machine maintenance, loading, unloading, transportation, mechanical restoration, assembly, installation and loading of parts, transfer of parts and blanks, laser and spot welding, laser cutting, metal casting, stamping, forging and bending, spray paint and enamel coating, packaging and picking, product quality control.

Industrial robots consist of a manipulator and a software control system for the manipulator's working body - a gripping device (gripper).

In industrial production, a huge number of specialized robotic manipulators of various companies are used: 3D Robotics, ABB Robotics, Aethon, Alphabet, Inc. (Google), Amazon, Autonomous Solutions (ASI), CANVAS Technology, Carbon Robotics, Clearpath Robotics, Cyberdyne, Delphi Automotive, DJI, Ekso Bionics, EPSON Robots, FANUC Robotics, Fetch Robotics, Foxconn Technologies Group, GreyOrange, IAM Robotics, Intuitive Surgical, iRobot, Jibo, Kawasaki Robotics, Knightscope, KUKA, Lockheed Martin, Locus Robotics, Omron Adept Technologies Inc., Open Bionics, Rethink Robotics, ReWalk Robotics, Robotiq, Samsung, Savioke, Schunk, Seegrid, Siasun Robot & Automation Co.Ltd., SoftBank Robotics Corporation, Soil Machine Dynamics Ltd., Swisslog, Titan Medical, Toyota, ULC Robotics, Universal Robotics, Inc., Vecna Technologies, Verb Surgical, VEX Robotics, Yamaha Robotics, Yaskawa, Arkodim, Bit Robotics, Exoathlete, Ronavi Robotics, Eidos Robotics. The international company KUKA produces dozens of industrial robotic manipulators used in various industries with a wide range of carrying power parameters and handling radius. For example, for the KR QUANTEC multipurpose industrial robotic arm, the payload is: 120 - 300 kg, and the maximum handling radius is 270 - 310 cm.

Let us consider the urgent task of improving automated production systems based on low-cost robotic manipulators with programmed control.

A lot of modern scientific works are devoted to the problems of robotics and automation of production.

In [1], robot reconfiguration technologies for automating production lines are developed and the concept of self-adjusting robot skills for production is proposed. Skills for performing various tasks obtained from the current instructions of the working wizard are transferred to industrial manipulators.

Known methods of elastogeometric calibration to improve the accuracy of positioning of industrial robots 6 DOF [2]. In the process of elastogeometric calibration, the optimal set of robot configurations and external loads is determined. Elastogeometric calibration reduces the maximum position error to 0.96 mm, which, in contrast to the kinematic calibration, reduces the error to 2.557 mm.

A methodology has been developed for planning and optimizing the movement of the robot, which provides a set of robot trajectories for each task while minimizing the cycle time [3].

In [4], the problem of controlling the accuracy of following a given trajectory for an industrial robot is researched. The considered approach is applied to the predictor controller for following the trajectory on the KUKA LWR IV robot.

The study [5] analyzes the safety of controllers and software vulnerabilities for industrial robots. Safety standards and safety problems in industrial robotics are considered.

For industrial robots, an algorithm for correcting the welding path in real time has been developed [6].

An intelligent algorithm for determining the trajectory of an industrial manipulator based on the analysis of alternative configurations while minimizing the operation time was proposed in [7].

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In the study [8], an adaptive control method was proposed for industrial robots with six degrees of freedom, which reduces errors with external noise and with parametric uncertainties.

In [9], a wireless three-dimensional manipulation system is developed on a tablet PC based on augmented reality. The system allows you to virtually control the movement of the robotic arm in three-dimensional space through augmented reality. An intelligent system is being developed to build a robot trajectory without collisions.

The study [10] proposed an improved A * algorithm for solving the robot path planning problem. Optimization of the local path between the current and the end node is applied, taking into account the safety of the path, the absence of collisions on the path and reduction of the path length.

In [11], to reduce the motion uncertainty of industrial robots with six degrees of freedom, a parameter identification method based on the Denavit-Hartenberg model, where excess parameters are taken into account during identification, is presented. For the kinematic model of an industrial robot with six degrees of freedom, the linearization method and an alternative identification algorithm with a modified least-squares scheme are used.

In [12], the problems of dynamic identification for an industrial robotic arm with six degrees of freedom are investigated and a procedure for identifying parameters based on a modified cuckoo search algorithm is presented. The nonlinear model of the robotic arm takes into account friction in the joints.

In [13], an adaptive robust controller is developed for dynamically tracking a robot trajectory with 6 degrees of freedom with parametric uncertainties and external noise. The controller is developed on the basis of a combination of sliding mode control, adaptive control and has the ability to adapt and resistance to parametric changes and uncertainties.

In the study [14], a smooth path planning method for industrial robots was proposed. The trajectory is planned for joint movement of three parts: accelerated, slowed down part and part with constant speed. In the accelerated part and the slowed part, the acceleration is represented by a fourth-order polynomial with the property of multiplicity of roots. The time-optimal trajectory is obtained by maximizing the part at a constant speed under kinematic constraints.

In [15], 6DOF smart industrial manipulators were studied and two compensation methods based on reinforced learning were presented. The study compares the effectiveness of learning-based methods with typical tracking controllers, proportional derivatives, predictive models, and iterative learning.

In the study [16], a new method is proposed for a dynamic model of a robotic arm with 6 degrees of freedom to increase the accuracy of movement. A centrosymmetric model of static friction is used to describe the hysteretic effect of friction in joints. The hybrid whale optimization algorithm and the genetic algorithm are used to determine the dynamic parameters of the 6 joints of the robot.

In [17], an error model for the kinematic calibration of a serial robot based on double quaternions was presented. Double quaternions are a spatial transformation and are used for kinematic analysis of the robot. The error model is obtained from direct kinematic equations using the algebra of double quaternions. The error model contains all the basic geometric parameters of the robot for kinematic calibration. The Denavit - Hartenberg error model in kinematic analysis gives an intuitive geometric meaning of kinematic parameters.

The study [18] presented a method for planning the trajectory to minimize synthesis errors and obtain stable motion of industrial robots. The minimal synthesis error is determined under kinematic and dynamic constraints. A kinematic study of the dynamic model of the robot ER3A-C60 is carried out taking into account the flexibility of all connections. The trajectory planning technique can improve the tracking accuracy of the final effector, while controlling the movement time and the requirements of continuous trajectory control.

Software-controlled robotic manipulators are widely used at various industrial enterprises without expensive sensors and artificial intelligence elements.

The paper presents a method for determining the spatial and kinematic characteristics of the working body of a manipulator based on the matrix method in the kinematics of robots and second-order Lagrange matrix equations in dynamics. The method allows you to determine in space the correct position of the working body of the manipulator. The method allows you to calculate the working optimal operating modes of the manipulator, increase the speed of operations on the production line. The presented matrix method was applied in [23] for the study of manipulators with five degrees of freedom.

This approach optimizes the number of universal manipulators with program control for various technological operations, and also allows to increase the productivity of a robotic production line while maintaining the quality of the process.

The method allows you to calculate and optimize the sequence of movements of the links of the robot-manipulator to move the grip of the manipulator to a given point in space. In this case, it is possible to calculate and control the speed and acceleration of the grip of the robot during movement, reducing the time of the duration of the tact of the robot.

II. Manipulator motion mode calculation method

To build a mathematical model of the dynamics of the robot-manipulator, we use the matrix method and matrix equations of the Lagrange of the second kind. In the method, we determine the transition matrices from one local coordinate system of one manipulator link to another local coordinate system of the next manipulator link. In accordance with [19], by a special choice of local coordinate systems, one can describe the transition from one local coordinate system of a link to another by means of four parameters, and not six as in general form. Four parameters correspond to rotation around the axis, two movements along the axes and subsequent rotation around the axis.

Imagine a method for calculating the optimal movement of the manipulator.

Let the manipulator under study have n movable links.

Let us introduce the notation of the beginning of the local coordinate system associated with the i - link of the manipulator O_i.

For a global coordinate system associated with a fixed coordinate system, we denote the origin O_0.

Denote the generalized coordinates for the manipulator by

$$q_i, (i = 1,...,n).$$

We introduce the extended radius vector for the point in the i-th local coordinate system:

$$R_i = \begin{bmatrix} x_i & y_i & z_i & 1 \end{bmatrix}^T.$$

To move from the initial coordinate system O_0 to the local coordinate system of the first link O_1 rotation around the axis by an angle α, two movements along the axes by a, b and a subsequent rotation around the axis by an angle β.

The transition matrix from one coordinate system to the next has the form::

$$A_{i,i+1} = \begin{bmatrix} \cos(\alpha) & -\cos(\beta)\sin(\alpha) & \sin(\alpha)\sin(\beta) & b\cos(\alpha) \\ \sin(\alpha) & \cos(\alpha)\cos(\beta) & -\cos(\alpha)\sin(\beta) & b\sin(\alpha) \\ 0 & \sin(\beta) & \cos(\beta) & a \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transition matrix $A_{i,i+1}$ connects the radii of the vectors of the coordinate systems i and i + 1 with the following formula:

$$R_i = A_{i,i+1} R_{i+1}$$

The transition matrix from the global coordinate system O_0 to the local coordinate system O_i is determined by the product of the transition matrices: $A_{0i} = A_{01} A_{12} \dots A_{i-1\ i}$

Thus, the position of the grip of the manipulator in the global coordinate system is determined.

The links of the manipulator are connected by cylindrical hinges and sliding joints, in which low friction can be neglected.

To compose the differential equations of motion of the manipulator, we apply the matrix Lagrange equations in the form:

$$LT + DP = Q,$$

where

$L = \left[\frac{d}{dt}\left(\frac{\partial T}{\partial q_1'}\right) - \frac{\partial T}{\partial q_1}, ..., \frac{d}{dt}\left(\frac{\partial T}{\partial q_i'}\right) - \frac{\partial T}{\partial q_i} \right]$ - is the line vector of the Lagrange operators, T is the total kinetic energy of all parts of the manipulator, Q is the vector line of the generalized forces created by the manipulator drives..

$DP = \left[\frac{\partial P}{\partial q_1}, ..., \frac{\partial P}{\partial q_i} \right]$ - is a line-vector of derivatives of potential energy.

We determine the kinetic energy for each link of the manipulator using transition matrices according to the formulas:

$$T_i = \frac{1}{2} tr\left(\dot{A}_{0i} H_i \dot{A}_{0i}^T \right),$$

where H_i is the link inertia matrix.

$$H_i = \begin{bmatrix} J_{ixx} & J_{ixy} & J_{ixz} & m_i x_{Ci} \\ J_{iyx} & J_{iyy} & J_{iyz} & m_i y_{Ci} \\ J_{izx} & J_{izy} & J_{izz} & m_i z_{Ci} \\ m_i x_{Ci} & m_i y_{Ci} & m_i z_{Ci} & m_i \end{bmatrix},$$

where m_i is the mass of the link,

$J_{ixx}, J_{iyy}, J_{izz}$ are components of the link inertia tensor relative to the axes.

The coordinates of the center of gravity of the link in the local coordinate system are determind as x_{Ci}, y_{Ci}, z_{Ci}.

The moments of inertia of the link relative to the axes are denoted by J_{xi}, J_{yi}, J_{zi}.

The equalities are valid for the axial moments of inertia:

$$J_{xi} = J_{iyy} + J_{izz}, J_{yi} = J_{ixx} + J_{izz}, J_{zi} = J_{ixx} + J_{iyy}.$$

Determine the total kinetic energy of all the links of the manipulator: $T = \sum_{i=1}^{n} T_i$.

We define the potential energy for the gravity of each link in the form: $P_i = -m_i G^T A_i C_i$,

where $C_i = \begin{bmatrix} x_{Ci} & y_{Ci} & z_{Ci} & 1 \end{bmatrix}^T$ – is the column of coordinates of the center of gravity of the link

$G_i = \begin{bmatrix} 0 & 0 & -g & 0 \end{bmatrix}^T$ – column of acceleration of gravity.

We determine the total potential energy of all the links of the manipulator: $P = \sum_{i=1}^{n} P_i$.

Substituting the total kinetic, potential energy and generalized forces for the link drives in the Lagrange matrix equation, we obtain a system of differential equations of motion for an industrial robot.

To formulate the Cauchy problem, we set the initial conditions for the generalized coordinates and speeds of the links of the manipulator for the system of nonlinear differential equations.

To construct solutions to a system of second-order nonlinear differential equations, the method of transformations was applied [20,22].

Having obtained the solution, we determine the generalized coordinates, velocities, and accelerations for the manipulator.

Having built an analytical solution for the system, we obtain the dependences for the generalized coordinates, velocities, and accelerations on the generalized forces of the manipulator electric drives. Using these dependencies, we determine the values of the generalized forces of the electric drives necessary to move the gripper to the end point.

Using the transition matrices $A_{i-1,i}$ and the extended radius of the coordinate vector $R_0 = A_{01}A_{12}\dots A_{k-1\ k}R_k$ we obtain the grip coordinates of the manipulator in the global coordinate system, depending on the generalized coordinates.

Substituting the generalized coordinates, we construct the trajectory of the gripper in time for given generalized forces of the manipulator electric drives.

To optimize the grip movement of the manipulator, we consider the law of uniformly variable grip movement at the required travel time t_k, the initial and final position of the grip of the form:

$$x_k = x_0 + v_{x0}t_k + w_x t_k^2, y_k = y_0 + v_{y0}t_k + w_y t_k^2, z_k = z_0 + v_{z0}t_k + w_z t_k^2,$$
$$v_x = v_{x0} + w_x t_k, v_y = v_{y0} + w_y t_k, v_z = v_{z0} + w_z t_k,$$

where (x_0, y_0, z_0), (x_k, y_k, z_k) is the initial and final coordinates of the tong, $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ is the speed of movement of the tong, $w = \sqrt{w_x^2 + w_y^2 + w_z^2}$ - is an acceleration of the tong. We take into account that with equal alternating motion, tangential acceleration is constant.

Having determined the solutions of the Lagrange differential equations, we find the dependences of the generalized coordinates, velocities, and accelerations, and applying the law of uniformly variable grip motion, we determine the optimal manipulator grip motion.

III. MODELING OF A MANIPULATOR WITH SIX DEGREES OF FREEDOM

Consider a multipurpose robotic manipulator with six degrees of freedom, which includes six links, a base, link drives, and a gripper.

The kinematic diagram of the robot is shown in Fig. 1 and consists of five rotational kinematic pairs and one translational pair. The robot model is applicable to industrial manipulators, the kinematic diagram of which is shown in Fig. 1.

When developing dynamic equations of the robot, we use the matrix method and matrix dynamic Lagrange equations. The links of the industrial manipulator are modeled by rods, the joints are modeled by cylindrical joints and sliding joints. We assume that the friction in the joints is small and is not taken into account when deriving the robot model.

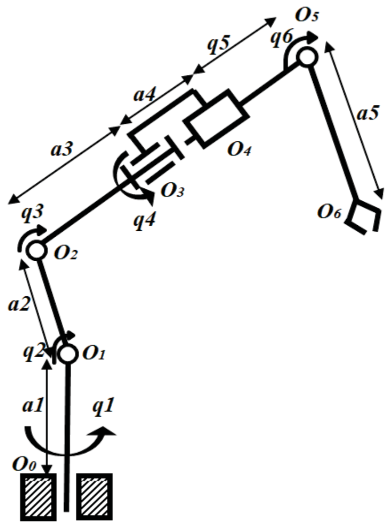

Fig. 1. The kinematic diagram of the manipulator

A three-dimensional model of an industrial manipulator was built and modeling was carried out in a specialized computer program System-Modeler (Fig. 2).

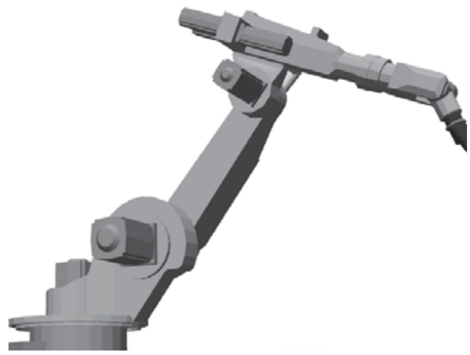

Fig. 2. Three-dimensional model of the manipulator

Define the coordinate system of the robot links at points $O_1, O_2, O_3, O_4, O_5, O_6$. The absolute coordinate system is connected with the fixed base of the manipulator at a point O_0.

Take as a generalized coordinates of the manipulator with six degrees of freedom - the angles of rotation of the links and the length of the extension of the arm. Here we measure angles in radians, lengths in centimeters.

We introduce the notation of periodic functions:

$$C_i = \cos[q_i(t)], S_i = \sin[q_i(t)],$$
$$\sin[2q_2(t)] = S_7, \sin[2(q_2(t) + q_3(t))] = S_8,$$
$$\sin[2q_2(t) + q_3(t)] = S_9, \cos[2q_2(t) + q_3(t)] = C_9 .$$
$$\cos[2q_2(t)] = C_7, \cos[2(q_2(t) + q_3(t))] = C_8,$$
$$\sin[q_2(t) + q_3(t)] = S_{10}, \cos[q_2(t) + q_3(t)] = C_{11}$$

The transition matrices from the absolute coordinate system to the coordinate system are defined:

$$A_{01}=\begin{bmatrix} C_1 & -S_1 & 0 & 0\\ S_1 & C_1 & 0 & 0\\ 0 & 0 & 1 & a_1\\ 0 & 0 & 0 & 1\end{bmatrix},\ A_{12}=\begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & C_2 & -S_2 & 0\\ 0 & S_2 & C_2 & a_2\\ 0 & 0 & 0 & 1\end{bmatrix},$$

$$A_{23}=\begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & C_3 & -S_3 & 0\\ 0 & S_3 & C_3 & a_3\\ 0 & 0 & 0 & 1\end{bmatrix},\ A_{34}=\begin{bmatrix} C_4 & -S_4 & 0 & 0\\ S_4 & C_4 & 0 & 0\\ 0 & 0 & 1 & a_4\\ 0 & 0 & 0 & 1\end{bmatrix},$$

$$A_{45}=\begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & q_5\\ 0 & 0 & 0 & 1\end{bmatrix},\ A_{56}=\begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & C_6 & -S_6 & 0\\ 0 & S_6 & C_6 & a_5\\ 0 & 0 & 0 & 1\end{bmatrix}.$$

The transition matrices from the absolute coordinate system O_0 to the coordinate system O_i are defined:

$$A_{0i}=A_{01}A_{12}\dots A_{i-1\ i}$$

$$A_{02}=\begin{bmatrix} C_1 & -C_2S_1 & S_1S_2 & 0\\ S_1 & C_1C_2 & -C_1S_2 & 0\\ 0 & S_2 & C_2 & a_1+a_2\\ 0 & 0 & 0 & 1\end{bmatrix},$$

$$A_{03}=\begin{bmatrix} C_1 & -C_2C_3S_1+S_1S_2S_3 & C_3S_1S_2+C_2S_1S_3 & a_3S_1S_2\\ S_1 & C_1C_2C_3-C_1S_2S_3 & -C_1C_3S_2-C_1C_2S_3 & -a_3C_1S_2\\ 0 & C_3S_2+C_2S_3 & C_2C_3-S_2S_3 & a_1+a_2+a_3C_2\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$A_{04}=\begin{bmatrix} C_1C_4+\left(-C_2C_3S_1+S_1S_2S_3\right)S_4 & C_4\left(-C_2C_3S_1+S_1S_2S_3\right)-C_1S_4 & C_3S_1S_2+C_2S_1S_3 & a_3S_1S_2+a_4\left(C_3S_1S_2+C_2S_1S_3\right)\\ C_4S_1+\left(C_1C_2C_3-C_1S_2S_3\right)S_4 & C_4\left(C_1C_2C_3-C_1S_2S_3\right)-S_1S_4 & -C_1C_3S_2-C_1C_2S_3 & -a_3C_1S_2+a_4\left(-C_1C_3S_2-C_1C_2S_3\right)\\ \left(C_3S_2+C_2S_3\right)S_4 & C_4\left(C_3S_2+C_2S_3\right) & C_2C_3-S_2S_3 & a_1+a_2+a_3C_2+a_4\left(C_2C_3-S_2S_3\right)\\ 0 & 0 & 0 & 1\end{bmatrix},$$

$$A_{05}=\begin{bmatrix} C_1C_4+\left(-C_2C_3S_1+S_1S_2S_3\right)S_4 & C_4\left(-C_2C_3S_1+S_1S_2S_3\right)-C_1S_4 & C_3S_1S_2+C_2S_1S_3 & a_3S_1S_2+a_4\left(C_3S_1S_2+C_2S_1S_3\right)+q_5\left(C_3S_1S_2+C_2S_1S_3\right)\\ C_4S_1+\left(C_1C_2C_3-C_1S_2S_3\right)S_4 & C_4\left(C_1C_2C_3-C_1S_2S_3\right)-S_1S_4 & -C_1C_3S_2-C_1C_2S_3 & -a_3C_1S_2+a_4\left(-C_1C_3S_2-C_1C_2S_3\right)+q_5\left(-C_1C_3S_2-C_1C_2S_3\right)\\ \left(C_3S_2+C_2S_3\right)S_4 & C_4\left(C_3S_2+C_2S_3\right) & C_2C_3-S_2S_3 & a_1+a_2+a_3C_2+a_4\left(C_2C_3-S_2S_3\right)+q_5\left(C_2C_3-S_2S_3\right)\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$A_{06}=\begin{bmatrix} C_1C_4+\left(-C_2C_3S_1+S_1S_2S_3\right)S_4 & C_6\left(C_4\left(-C_2C_3S_1+S_1S_2S_3\right)-C_1S_4\right)+\left(C_3S_1S_2+C_2S_1S_3\right)S_6 & C_6\left(C_3S_1S_2+C_2S_1S_3\right)-\left(C_4\left(-C_2C_3S_1+S_1S_2S_3\right)-C_1S_4\right)S_6 & a_3S_1S_2+a_4\left(C_3S_1S_2+C_2S_1S_3\right)+a_5\left(C_3S_1S_2+C_2S_1S_3\right)+q_5\left(C_3S_1S_2+C_2S_1S_3\right)\\ C_4S_1+\left(C_1C_2C_3-C_1S_2S_3\right)S_4 & C_6\left(C_4\left(C_1C_2C_3-C_1S_2S_3\right)-S_1S_4\right)+\left(-C_1C_3S_2-C_1C_2S_3\right)S_6 & C_6\left(-C_1C_3S_2-C_1C_2S_3\right)-\left(C_4\left(C_1C_2C_3-C_1S_2S_3\right)-S_1S_4\right)S_6 & -a_3C_1S_2+a_4\left(-C_1C_3S_2-C_1C_2S_3\right)+a_5\left(-C_1C_3S_2-C_1C_2S_3\right)+q_5\left(-C_1C_3S_2-C_1C_2S_3\right)\\ \left(C_3S_2+C_2S_3\right)S_4 & C_4C_6\left(C_3S_2+C_2S_3\right)+\left(C_2C_3-S_2S_3\right)S_6 & C_6\left(C_2C_3-S_2S_3\right)-C_4\left(C_3S_2+C_2S_3\right)S_6 & a_1+a_2+a_3C_2+a_4\left(C_2C_3-S_2S_3\right)+a_5\left(C_2C_3-S_2S_3\right)+q_5\left(C_2C_3-S_2S_3\right)\\ 0 & 0 & 0 & 1\end{bmatrix}$$

The coordinates of the grip of the manipulator in the absolute coordinate system O_0 are determined as a function of the generalized coordinates of the manipulator

$$x_{06}=a_3S_1S_2+a_4\left(C_3S_1S_2+C_2S_1S_3\right)+ a_5\left(C_3S_1S_2+C_2S_1S_3\right)+q_5\left(C_3S_1S_2+C_2S_1S_3\right)$$

$$y_{06}=-a_3C_1S_2+a_4\left(-C_1C_3S_2-C_1C_2S_3\right)+ a_5\left(-C_1C_3S_2-C_1C_2S_3\right)+q_5\left(-C_1C_3S_2-C_1C_2S_3\right)$$

$$z_{06}=a_1+a_2+a_3C_2+a_4\left(C_2C_3-S_2S_3\right)+ a_5\left(C_2C_3-S_2S_3\right)+q_5\left(C_2C_3-S_2S_3\right)$$

The total kinetic energy of all the links of the manipulator is determined: $T=\sum_{i=1}^{6}T_i$

The total potential energy of all the links of the manipulator is determined:

$$P=\sum_{i=1}^{6}P_i=g(a_2\sum_{i=2}^{6}m_i+a_1\sum_{i=1}^{6}m_i+C_2(a_3\sum_{i=3}^{6}m_i+ C_3\left(a_5m_6+a_4\sum_{i=4}^{6}m_i+\sum_{i=5}^{6}m_iq_5\right))-\left(a_5m_6+a_4\sum_{i=4}^{6}m_i+\sum_{i=5}^{6}m_iq_5\right)S_2S_3)$$

Substituting the kinetic, potential energy and generalized forces into the Lagrange equations, we obtain a system of equations of motion for a manipulator with six degrees of freedom.

$$
\begin{array}{l}
q_1'((k_5+k_6q_5+k_{56}q_5^2)S_8(q_2'+q_3'))+q_1'((1-C_8)(k_5+k_{56}q_5)q_5')+ \\
(0.5k_{15}+0.5(1-C_8)k_{56}q_5^2+m_6(0.5a_5^2(1-C_8)+0.5i_6^2+a_5(1-C_8)q_5))q_1''+ \\
a_4^2k_{46}(S_8q_1'(q_2'+q_3')+0.5(1-C_8)q_1'')+ \\
a_3^2(k_{36}S_7q_1'q_2'+(0.5(1-C_7)m_3+0.5k_{46}-0.5C_7k_{46})q_1'')+ \\
a_4(q_1'((2k_5+2k_{56}q_5)S_8(q_2'+q_3')+(1-C_8)k_{56}q_5')+ \\
(1-C_8)(k_5+k_{56}q_5)q_1'')+ \\
a_3(q_1'((2k_5+2k_{56}q_5)S_9q_2'+(k_5+k_{56}q_5)(-S_3+S_9)q_3'+ \\
(-C_9k_{56}+C_3k_{56})q_5')+(a_5(C_3-C_9)m_6+(-C_9k_{56}+C_3k_{56})q_5)q_1''+ \\
a_4(q_1'(2k_{46}S_9q_2'+k_{46}(-S_3+S_9)q_3')+(-C_9k_{46}+C_3k_{46})q_1''))=Q_1
\end{array}
$$

$$
\begin{array}{l}
g(-k_5-k_{56}q_5)S_{10}+(-0.5a_5^2m_6-k_5q_5-0.5k_{56}q_5^2)S_8(q_1')^2+ \\
(2k_5+2k_{56}q_5)q_2'q_5'+(0.5k_{25}+k_{56}q_5^2+m_6(a_5^2+0.5i_6^2+2a_5q_5))q_2''+ \\
a_3^2(-0.5k_{36}S_7(q_1')^2+k_{36}q_2'')+a_4^2(-0.5k_{46}S_8(q_1')^2+k_{46}q_2'')+ \\
a_3(-gk_{36}S_2-a_4k_{56}S_9(q_1')^2-a_5m_6S_9(q_1')^2-k_{56}q_5S_9(q_1')^2- \\
2a_4k_{56}S_3q_2'q_3'-2k_{46}S_3q_2'q_3'-2k_{56}q_5S_3q_2'q_3'+ \\
m_4\left(-gS_2+a_4\left(-S_9(q_1')^2-2S_3q_2'q_3'\right)\right)+ \\
2C_3k_{56}q_2'q_5'+2C_3(a_5m_6+a_4k_{46}+k_{56}q_5)q_2'')+ \\
a_4(-gm_4S_{10}+m_6(-gS_{10}+(-a_5-q_5)S_8(q_1')^2+2q_2'q_5')+ \\
m_5\left(-gS_{10}-q_5S_8(q_1')^2+2q_2'q_5'\right)+(2a_5m_6+2k_{56}q_5)q_2'')=Q_2
\end{array}
$$

$$
\begin{array}{l}
(m_5+m_6)q_5(-gS_{10}-0.5q_5S_8(q_1')^2+ \\
a_3\left(-0.5S_9(q_1')^2+S_3\left(0.5(q_1')^2+(q_2')^2\right)\right)+2q_3'q_5')+ \\
\left(0.5i_3^2m_3+0.5i_4^2m_4+0.5i_5^2m_5+0.5i_6^2m_6+(m_5+m_6)q_5^2\right)q_3''+ \\
a_5^2m_6\left(-0.5S_8(q_1')^2+q_3''\right)+ \\
a_4^2\left((-0.5m_4-0.5m_5-0.5m_6)S_8(q_1')^2+(m_4+m_5+m_6)q_3''\right)+ \\
a_4(m_4\left(-gS_{10}+a_3\left(-0.5S_9(q_1')^2+S_3\left(0.5(q_1')^2+(q_2')^2\right)\right)\right)+ \\
m_6(-gS_{10}+(-a_5-q_5)S_8(q_1')^2+ \\
a_3\left(-0.5S_9(q_1')^2+S_3\left(0.5(q_1')^2+(q_2')^2\right)\right)+2q_3'q_5')+ \\
m_5(-gS_{10}-q_5S_8(q_1')^2+a_3\left(-0.5S_9(q_1')^2+S_3\left(0.5(q_1')^2+(q_2')^2\right)\right)+ \\
2q_3'q_5')+2\left(a_5m_6+(m_5+m_6)q_5\right)q_3'')+ \\
a_5m_6(-gS_{10}+a_3\left(-0.5S_9(q_1')^2+S_3\left(0.5(q_1')^2+(q_2')^2\right)\right)+ \\
2q_3'q_5'+q_5\left(-S_8(q_1')^2+2q_3''\right))=Q_3
\end{array}
$$

$$
0.5q_4''\left(i_4^2m_4+i_5^2m_5+i_6^2m_6\right)=Q_4
$$

$$
\begin{array}{l}
m_6(gC_{11}+(0.5k_{45}(-1+C_8)+ \\
a_3 0.5(-C_3+C_9)+0.5(-1+C_8)q_5)(q_1')^2+ \\
(-k_{45}-a_3C_3-q_5)(q_2')^2+(-k_{45}-q_5)(q_3')^2+q_5'')+ \\
m_5(gC_{11}+a_3\left(0.5C_9(q_1')^2+C_3\left(-0.5(q_1')^2-(q_2')^2\right)\right)+ \\
a_4\left(0.5(-1+C_8)(q_1')^2-(q_2')^2-(q_3')^2\right)+ \\
q_5\left(0.5(-1+C_8)(q_1')^2-(q_2')^2-(q_3')^2\right)+q_5'')=Q_5
\end{array}
$$

$$
0.5i_6^2m_6q_6''=Q_6
$$

The fourth and sixth equations of the system are easily integrated:

$$
q_4(t,Q)=\frac{t^2Q_4}{i_4^2m_4+i_5^2m_5+i_6^2m_6},\quad q_6(t,Q)=\frac{t^2Q_6}{i_6^2m_6}.
$$

To solve the remaining four differential equations of the system, we apply the polynomial transformation method [20-23] with the following parameters:

$$
m_1=200\ ,m_2=100\ ,m_3=30\ ,m_4=20\ ,m_5=10\ ,m_6=30\ ,
$$

$$
Q_1=100, Q_2=10, Q_3=10000,\ \ Q_4=0.1, Q_5=0.1, Q_6=0.01,
$$

$$
a_1=30\ ,a_2=50\ ,a_3=30\ ,a_4=20\ ,a_5=20\ ,a_6=10\ .
$$

Initial Values:

$$
\begin{array}{l}
q_1(0)=0, q_2(0)=3.14, q_3(0)=4.71, q_4(0)=0, \\
q_5(0)=0, q_6(0)=3.14, q_i'(0)=0
\end{array}
$$

The transformation method [20] allows you to find a solution with all non-linear components of the original system. The transformation method allows you to build a solution of a nonlinear system of differential equations in an analytical form with numerical coefficients.

The solution of the system of three differential equations was obtained by the method of transformations.

$$
q_1(t)=-0.005-0.003t+0.007t^2,
$$

$$
q_2(t)=3.1+0.128t-0.011t^2,
$$

$$
q_3(t)=4.507+0.588t-0.049t^2,
$$

$$
q_4(t)=0.167t^2,
$$

$$
q_5(t)=2.298-10.948t+8.342t^2,
$$

$$
q_6(t)=0.239t^2\ .
$$

For industrial manipulators with a kinematic diagram shown in Fig. 2, the kinematic characteristics are calculated for the values of the mass parameters and link lengths of the industrial manipulator.

Model verification of an industrial manipulator is carried out by parallel modeling in a specialized computer program.

Fig. 3 shows the rotation angles (in radians) for the links of the manipulator.

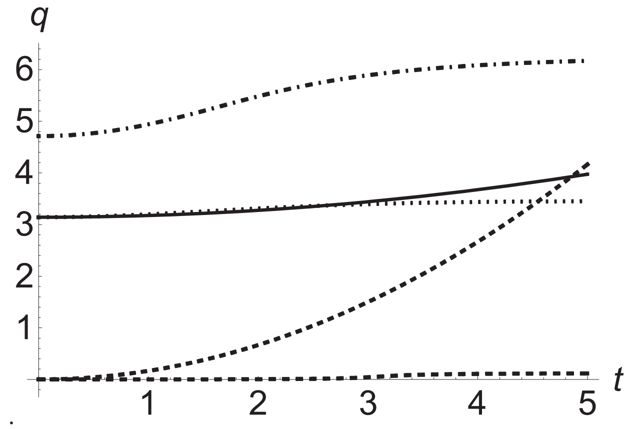

Fig. 3. The rotation angles (in radians) of the manipulator links

Fig. 4 shows the extension of the arm of the manipulator (in centimeters).

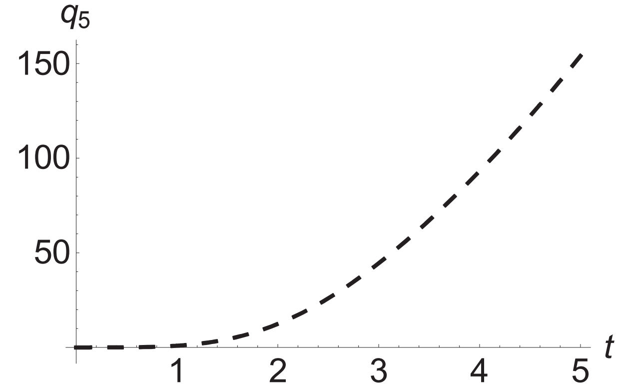

Fig. 4. Extension of the arm of the manipulator (in centimeters)

Fig. 5 shows the generalized speeds for the manipulator.

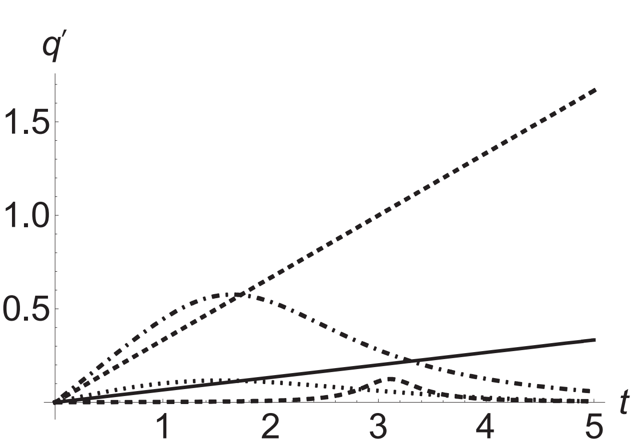

Fig. 5. The speed of the links of the manipulator

In figures 3,4,5 the following legend is used:

----- q_1 ······· q_2 -·-·- q_3

------ q_4 — — q_5 — q_6

Substituting the coordinates of the manipulator gripper into the equations of equal alternating movement of the manipulator grip, we obtain algebraic equations for the optimal manipulator grip motion in time. t_k .

$$S_1\left(a_3S_2+\left(a_4+a_5+q_5\right)\left(C_3S_2+C_2S_3\right)\right)-v_{x0}t_k-w_xt_k^2=0,$$

$$-C_1\left(a_3S_2+\left(a_4+a_5+q_5\right)\left(C_3S_2+C_2S_3\right)\right)-v_{y0}t_k-w_yt_k^2=0,$$

$$a_1+a_2+a_3C_2+\left(a_4+a_5+q_5\right)\left(C_2C_3-S_2S_3\right)-v_zt_k-w_zt_k^2=0,$$

Simplify the system of three equations, taking into account the notation $C_i=\mathrm{Cos}\left[q_i\left(t,Q\right)\right],\ S_i=\mathrm{Sin}\left[q_i\left(t,Q\right)\right]$:

$$\mathrm{Sin}[q_1]\left(\mathrm{Sin}[q_2]a_3+\mathrm{Sin}[q_2+q_3]\left(a_4+a_5+q_5\right)\right)-v_{x0}t_k-w_xt_k^2=0$$

$$-\mathrm{Cos}[q_1]\left(\mathrm{Sin}[q_2]a_3+\mathrm{Sin}[q_2+q_3]\left(a_4+a_5+q_5\right)\right)-v_{y0}t_k-w_yt_k^2=0$$

$$a_1+a_2+\mathrm{Cos}[q_2]a_3+\mathrm{Cos}[q_2+q_3]\left(a_4+a_5+q_5\right)-v_{z0}t_k-w_zt_k^2=0$$

An analytical solution of the system of equations is obtained

$$q_1\left(t\right)=\frac{1}{2}\mathrm{ArcCos}\left[\frac{\left(v_{y0}t_k+w_yt_k^2\right)^2-\left(v_{x0}t_k+w_xt_k^2\right)^2}{\left(v_{y0}t_k+w_yt_k^2\right)^2+\left(v_{x0}t_k+w_xt_k^2\right)^2}\right]$$

$$q_2\left(t\right)=\mathrm{ArcTan}\Big[\left(a_3^3P_{\mathrm{za}}+a_3P_{\mathrm{za}}\left(-a_6^2+P_{\mathrm{xy}}^2+P_{\mathrm{za}}^2\right)\right)/P_{\mathrm{xa}}-$$
$$\sqrt{-a_3^2P_{\mathrm{xy}}^2\left(a_3^4+\left(-a_6^2+P_{\mathrm{xy}}^2+P_{\mathrm{za}}^2\right)^2-2a_3^2\left(a_6^2+P_{\mathrm{xy}}^2+P_{\mathrm{za}}^2\right)\right)}/P_{\mathrm{xa}}\Big]$$

$$q_3\left(t\right)=-\mathrm{ArcTan}[\frac{1}{a_3^2a_6\left(P_{\mathrm{xy}}^2+P_{\mathrm{za}}^2\right)^2}(a_3^5\left(P_{\mathrm{xy}}^2-P_{\mathrm{za}}^2\right)+$$
$$a_3a_6^2\left(a_6^2-P_{\mathrm{xy}}^2-P_{\mathrm{za}}^2\right)\left(P_{\mathrm{xy}}^2-P_{\mathrm{za}}^2\right)-a_3^3\left(P_{\mathrm{xy}}^2-P_{\mathrm{za}}^2\right)\left(2a_6^2+P_{\mathrm{xy}}^2+P_{\mathrm{za}}^2\right)+$$
$$2(a_3^2-a_6^2)P_{\mathrm{za}}\sqrt{-a_3^2P_{\mathrm{xy}}^2\left(a_3^4+\left(-a_6^2+P_{\mathrm{xy}}^2+P_{\mathrm{za}}^2\right)^2-2a_3^2\left(a_6^2+P_{\mathrm{xy}}^2+P_{\mathrm{za}}^2\right)\right)})]$$

$$a_6=a_4+a_5+\frac{t^2Q_5}{i_5^2m_5+i_6^2m_6},P_{\mathrm{za}}=v_{z0}t_k+w_zt_k^2-a_1-a_2,$$

$$P_{\mathrm{xy}}=\left(v_{y0}t_k+w_yt_k^2\right)^2+\left(v_{x0}t_k+w_xt_k^2\right)^2,P_{\mathrm{xa}}=a_3^2\left(P_{\mathrm{xy}}^2+P_{\mathrm{za}}^2\right)$$

The analytical solution for $q_4\left(t\right),q_5\left(t\right),q_6\left(t\right)$ is obtained from the matrix dynamic equations of Lagrange.

$$q_4(t)=(t^2Q_4)/(i_4^2m_4+i_5^2m_5+i_6^2m_6),$$
$$q_5(t)=(t^2Q_5)/(i_5^2m_5+i_6^2m_6),$$
$$q_6(t)=(t^2Q_6)/(i_6^2m_6).$$

Thus, the functions of the generalized coordinates of the gripper of the manipulator are determined for uniform movement of the gripper.

Fig. 6 shows the trajectories of the tong along the x, y, z axes relative to the fixed base of the rack..

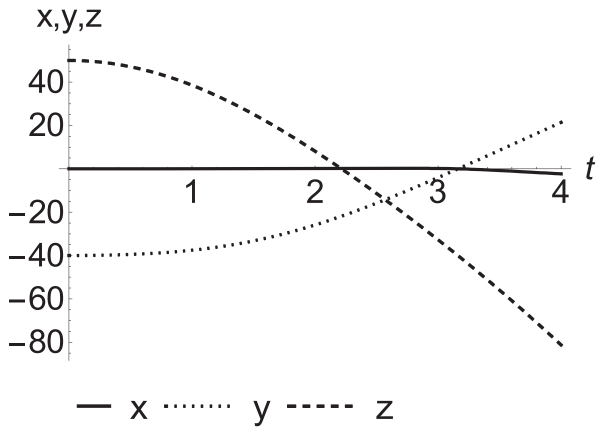

Fig. 6. Manipulator grip coordinates

Fig. 7 shows the optimal spatial trajectory of the gripper relative to the fixed base of the manipulator strut..

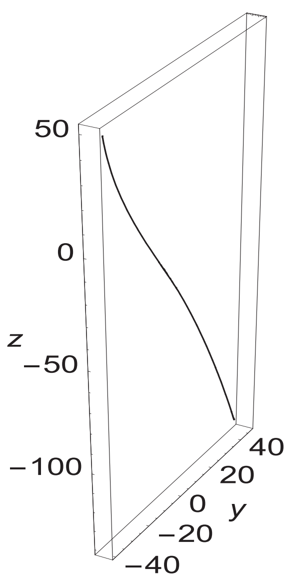

Fig. 7. The spatial trajectory of the grip of the manipulator

The presented approach allows us to construct an optimal spatial trajectory of the uniform motion of the manipulator's grip. All mathematical calculations were checked in specialized computer mathematical packages.

IV. CONCLUSION

The work considers the problem of improving automated production systems based on low-cost robotic manipulators with programmed control. For widespread use at various industrial enterprises of robots-manipulators with programmed control without expensive sensors and elements of artificial intelligence, we use methods for determining the spatial and kinematic characteristics of the working body of the manipulator. The method for determining the kinematic characteristics is based on the matrix method in the kinematics of robots and the second-order Lagrange matrix equations in dynamics. The method allows you to calculate the working optimal operating modes of the manipulator, increase the speed of operations on the production line. A kinematic diagram of a manipulator with six degrees of freedom has been developed, and a mathematical model of the manipulator has been investigated using the matrix method. As a result of the analysis of the mathematical model, the coordinates, speeds and accelerations of the manipulator links are determined.

The presented approach optimizes the number of universal manipulators with program control for various technological operations, and also allows to increase the productivity of a robotic production line.

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