Time-Dependent Multiple Depot Vehicle Routing Problem on Megapolis Network under Wardrop's Traffic Flow Assignment

Alexander V. Mugayskikh, Victor V. Zakharov
Saint-Petersburg State University
Saint-Petersburg, Russia
alexander.mugaiskih@gmail.com, v.zaharov@spbu.ru

Tero Tuovinen
University of Jyväskylä
Jyväskylä, Finland
tero.tuovinen@jyu.fi

***Abstract*—In this work multiple depot vehicle routing problem is considered in case of variable travel times between nodes on a metropolis network. This variant of the classic multiple depot vehicle routing problem is motivated by the fact that in urban contexts variable traffic conditions play an essential role and can not be ignored in order to perform a realistic optimization. Time-travel matrices corresponding to each period of planning horizon were formed by solving the traffic assignment problem in conjunction with shortest path problem. Routing problem instances include from 20 to 100 customers randomly chosen from a road network of Saint-Petersburg. The results demonstrate that taking into account traffic flow information can reduce route time by 8-37% depending on number of customers in the problem instance.**

I. Introduction

The present article is devoted to possible ways of reducing costs of freight forwarding companies by taking into account road networks traffic load while planning delivery route. Basing upon research results of Russian leading specialists one can draw a conclusion that constantly increasing level of traffic congestion is becoming as essential reason of economic losses. Shorter transportation time would make transportation more efficient and increase the probability of improving the service level.

Routing problems on a megapolis network can be solved by constructing routes in terms of mathematical modelling. Generated routing plans will provide the minimum travel time and shortest-time path for vehicles travelling between given depot and customers at different times of the day. The main obstacle in applying such methods is that most of the models assume that the travel speeds are constant, and ignore the fact that travel speeds can change throughout the day. But in practice, solutions become not optimal, and non-feasible in some cases. It causes late arrivals at customers and additional hiring costs for the truck drivers.

Proposed in this article approach of planning delivery routes can reduce costs as it considers traffic information and avoids well-predictable traffic congestion in off-line vehicle routing. We focus on delays caused by traffic congestion during peak hours since they constitute a large part (from 70 up to 87%) of all traffic congestion delays [1].

We consider both traffic assignment and vehicle routing problems. By modelling traffic flow assignment on road networks, travel time matrices for 7 periods of a day are formed. Information on travel times is be used while constructing routes in the time-dependent variant of multiple depot vehicle routing problem. Road network of the Saint Petersburg is considered as an example to test the impact of our approach and show that the time-dependent model provides significant improvements over the model with fixed travel times.

The rest of the paper is organised as follows. In the next section, there is a literature review on time dependent models in vehicle routing. Sections 3 is devoted to description of general model of the time-dependent multiple depot vehicle routing problem (TD-MDVRP). We briefly describe Wardrop's principles of equilibrium assignment of traffic flows on road network and consider the formulation of the traffic assignment problem (TAP) in section 4. Section 5 reports experimental results on the road network of Saint-Petersburg. Firstly, the traffic assignment problem is solved by Frank-Wolf algorithm and travel time matrices are obtained by Dijkstra's algorithm. Secondly, we consider randomly generated TD-MDVRP instances and demonstrate the effectiveness of time-dependent approach in vehicle routing in comparison with static formulation. Section 6 concludes research and proposes future avenues of our study.

II. Literature review

Time Dependent Vehicle Routing problem (TDVRP) is the variant of the classic Vehicle Routing Problem (VRP) motivated by the fact that in a congested urban environment variable traffic conditions play an essential role and should not be ignored in order to perform a realistic optimization. Vehicle routing problem consists in finding a set of routes for identical vehicles based at the depot, such that each of the customers is visited exactly once minimizing the total routing cost. Since the introduction of VRP in work [2] developing real life applications of the routing problems have led to the emergence of a wide range of VRP flavors. This paper is focused on problems in which speeds are not constant and the travel time between two points is not a function of only the distance travelled.

Time dependent vehicle routing problems have received considerably little attention among researchers. Although these problems represent an urban congested environment more accurately than do their nontemporal counterparts. The time dependent vehicle routing problem formulation was first introduced in study [3]. Randomly generated small-sized instances were solved by nearest neighbour heuristics for the time-dependent vehicle routing problem without time windows. Travel times were represented by step functions of two or three time periods and defined by uniform distribution for each period. In work [4] authors developed a restricted dynamic programming approach based on heuristic algorithm for solving the time-dependent traveling salesman problem. The algorithm was extended to handle with a special case of travel time step functions for which the principle of optimality holds, i.e. partial path of minimum arrival time necessarily leads to a minimum tour.

Authors in [5] formulated node-based time dependent vehicle routing problem. In this formulation, constant speed r is assigned to each location for each time period. Thus, r_{ijt} is an average travel speed for a move from i to j starting at period t. But in fact such definitions as time-dependent travel time and time-dependent travel speed are equivalent since it is always possible to deduce these values from each other. The time-dependent travel speed model has been validated in a vehicle scheduling package used to schedule bank couriers in a number of large metropolitan cities in the United States, but no details are provided in the article.

The major weakness of the above models is that they do not satisfy the FIFO property firstly mentioned in [6]. This is intuitive and desirable property that guarantees if a vehicle leaves a node i for a node j at a given time t, any identical vehicle with the same destination leaving customer i at a time $t + e$, where $e > 0$, will always arrive later. To overcome this weakness, authors in study [4] forced vehicles to wait at a node to smooth the travel time function. But this suggestion can hardly ever be carried out in real-life applications. In work [6], the scheduling horizon is divided into three time periods and the travel times change from one period to the next. Travel speed is defined by continuous linear function. This approach can not correctly represent peaks sharpness because travel speed is supposed to linearly vary. To avoid this inconvenient, polynomial travel time functions were introduced in [7]. Multi-start metaheuristic procedure developed in the latter study is applied on Torino road network. Intelligent Transport Systems and Infomobility society provided data with the current speed and real time information on traffic congestion on the main streets of Torino with 5 minutes time intervals. Example presented in [7] deals only with ten points from Torino network.

Authors in work [8] report on computational tests with the travel time data obtained from the traffic information system LISB of the city Berlin. The travel time data are available for every edge of the road network and 214 five-minute intervals. Real-life time-dependent vehicle routing problems were solved by proposed approach based on savings, sequential insertion and 2-opt local search algorithms. In study [9] multi ant colony system method for TDVRPTW is illustrated in details. Authors define travel times on the arcs by step-like speed distribution which induces a partition of the time into several periods. Presented algorithm was tested on the Padua road network. The data collected by the automated traffic control system Cartesio consists in a set of 1522 nodes and 2579 arcs. Measurements of traffic data were held for all arcs in rush hours and for 50 of these, every hour.

A case study from a well-known 3C warehousing company in Taiwan resulted in 22% improvement over the companys current strategy as it reported in [10]. Problem instance included a distribution center located in the south of Taiwan with the aim to satisfy the requests from 25 nearby retail stores. Five time intervals in a day were defined for time-dependent modelling. According to the experimental results, the proposed method produces better vehicle routing with shorter transportation time but longer transportation distances. However, longer transportation distances would increase fuel cost. As a result, authors suggested to consider a tradeoff between operation time and transportation distance for future research.

The benefits of time-dependent vehicle routing and scheduling systems were also demonstrated in [11] by using real-world data for the road network in the north west of England. In the UK, the ITIS Floating Vehicle Data provides a national road network monitoring system. The system can be used to update journey times based on current road conditions, but also provides a record of past conditions so that travel times can be related to the time of the day, the day of the week and the season of the year. The case study in [12] was based on the distribution system of an electrical goods wholesaler. For its operation in the South West of the UK, the number of customers served per day ranged between 40 and 64 and the number of vans required was normally up to seven.

Recent theoretical studies of TD routing problems are focused on new models, including pollution control [12], [13], pickup and delivery [14], inventory management [15] and applying new algorithms such as a hybrid ant colony/tabu search algorithm [14], an adaptive genetic algorithm [15] or a local search with the time improvement phase [16]. A stand-alone review on time-dependent routing problems could be found in [17].

Only some papers handle with well-known Solomon's benchmark instances introduced in [18]. In study [6] authors do not consider capacity constraints and fleet size from Solomon instances. Solomon instances are also reported in [9], nevertheless, the results cannot be compared with previous ones since a different time speed function is used. Authors in [16] formulated new test problems based on Solomon instances that capture the typical speed variations of congested urban settings. The comparison of results on Solomon instances could be found in [19]. The authors used genetic algorithm for constructing routes.

III. MD-TDVRP FORMULATION

Let us consider oriented graph $G = (V, E)$, where $V = \{1, ..., N + M\}$ is the nodes array, and $E = \{(i, j) \mid i, j \in V\}$ is the arcs array. The nodes array consists of two arrays: V_{cust} represent the customers, V_{depot} stands for the array of depots and $V = V_{cust} \cup V_{depot}$, where $V_{cust} = \{1, ..., N\}$ and $V_{depot} = \{1, ..., M\}$.

Let us consider the set of heterogeneous vehicles $K = \{1, ..., L\}$. Assume, each vehicle could perform one trip to deliver goods to a number customers, but total amount of these goods could not exceed the vehicle capacity Q_k.

Each customer $j \in V_{cust}$ has a demand level d_j that should be met. We assume that each vehicle visit should fully satisfy the demand of the customer, so each customer could be visited only once. We also assume that there are enough stock in the depot to satisfy all demand.

We consider the time-dependent version of the routing problem, which means that, first, we take into account time durations of routing processes, and second, arc travel times might change during the day.

Each arc $(i, j) \in E$ is associated with travel time function $t_{ij}(b_i)$ of departure time b_i from node i. We consider departure times as variables. We use the notation $b_i, i \in V_{cust}$ for the departure times from customers, and b_0^k for the start time of vehicle $k \in K$.

The goal of optimization is to find a routing plan for vehicles (set of routes, or visiting sequences), that would fully satisfy demands of the customers, would not violate capacity restrictions and be the fastest, i.e. have the least possible total travel time.

Above formulated problem could be formalized in the form of integer programming problem. Let $\{x_{ij}\}_{i,j \in V}$ be a binary variable, that equals one, if customer i is directly followed by customer j in the routing plan, and zero otherwise. Let $\{y_{ik}\}_{i \in V, k \in K}$ be a binary variable, that equals one, if customer i is visited by vehicle k, and zero otherwise. Thus, we now present an integer linear programming formulation of the MD-TDVRP:

$$\sum_{i \in V} \sum_{j \in V} t_{ij}(b_i) x_{ij} \to min, \tag{1}$$

subject to:

$$\sum_{i \in V} x_{ij} = \sum_{i \in V} x_{ji} \qquad \forall j \in V; \tag{2}$$

$$\sum_{i \in V} x_{ij} = 1 \qquad \forall j \in V_{cust}; \tag{3}$$

$$\sum_{i \in V_{cust}} d_i y_{ik} \le Q_k \qquad \forall k \in K; \tag{4}$$

$$b_j \ge \sum_{i \in V_{cust}} x_{ij}(b_i + t_{ij}(b_i)) + x_{0j} \sum_{k \in K} y_{jv}(b_0^k + t_{ij}(b_0^k)), \qquad \forall j \in V_{cust} \tag{5}$$

Target function (1) represents the total travel time by all routes. Constraints (2) express the continuity property of the routes. Constraints (3) ensure that each customer visited exactly ones. Constraints (4) guarantee capacity restrictions to be held. The last group of constraints (5) represent low boundaries for departure time from each customer.

As it was mentioned earlier, b_i, $i \in V$ stands for the departure time from customer i. We assume that continuous piecewise function is defined as

$$t_{ij}(b_i) = \begin{cases} c_{ij}^h, & z_{h-1} + \triangle_{ij,h-1} \le b_i \le z_h - \triangle_{ijh} \\ c_{ij}^h + \frac{(c_{ij}^{h+1} - c_{ij}^h)(b_i - z_h + \triangle_{ijh})}{2\triangle_{ijh}}, & \\ z_h - \triangle_{ijh} < b_i < z_h + \triangle_{ijh}, & \end{cases} \tag{6}$$

where $h = 1, \dots, H$; $\triangle_{ij0} = \triangle_{ijH} = 0$.

This function firstly was introduced in work [8].

IV. Traffic assignment problem

The traffic assignment problem, in short TAP, consists in determining which routes to assign to the drivers who travel on a transportation network from some origins and some destinations. It is known that any traffic system holds user equilibrium after some time. In 1952, Wardrop introduced two principles that formalize different notions of network equilibrium [20]. The first principle states that the journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route. It explains that after the moment when network became equilibrium, no user has any incentive to change it's path choice because it would increase their travel costs in the network. The second principle states that the assignment should minimize the total travel times of all users. In the first case the assignment is named a user equilibrium, and, in the second case, one talks of system optimum. More detailed information on the user equilibrium problem (flow assignment with equal journey time by alternative routes) and system optimum (flow assignment with minimal average journey time) one can find in study [21]. We will consider user equilibrium as it is the result of a dynamical process of drivers route choice behavior.

Let $G = (N, A)$ be an oriented graph, where N is the set of nodes and A is the set of arcs. We denote R the set of origin nodes and S the set of destination nodes. We introduce K_{rs} which stands for the set of routes between origin node $r \in R$ and destination node $s \in S$, x_a — route flow on arc $a \in A$; d_a — travel time on $a \in A$. We denote the link flow on route $k \in K_{rs}$ as f_k^{rs} and total link flow between $r \in R$ and $s \in S$ — F^{rs}.

We assume boolean function which indicates if arc a is included into the route $k \in K_{rs}$:

$$\delta_{ak}^{rs} = \begin{cases} 1, & \text{if arc a } a \text{ is in route } k \in K_{rs}, \\ 0, & \text{else.} \end{cases}$$

The user optimum equilibrium can be found by solving the following nonlinear programming problem

$$\min_x \sum_{a \in A} \int_0^{x_a} d_a(u) du, \tag{7}$$

subject to

$$\sum_{k \in K_{rs}} f_k^{rs} = F^{rs}, \quad \forall r \in R, \, s \in S, \tag{8}$$

$$f_k^{rs} \geqslant 0, \quad \forall k \in K^{rs}, \, r \in R, \, s \in S, \tag{9}$$

$$x_a = \sum_{r \in R} \sum_{s \in S} \sum_{k \in K_{rs}} f_k^{rs} \delta_{a,k}^{rs}, \quad \forall a \in A. \tag{10}$$

where $d_a(f)$ — delay function of volume f on arc $a \in A$.

Many different types of volume-delay functions have been proposed and used in practice. However, the most widely used volume-delay functions is BRP-function (Bureau of Public Roads) [22] which is defined as

$$d_i(f_i) = t_i^0 \left(1 + \alpha_i \left(\frac{f_i}{p_i}\right)^{\beta_i}\right),$$

where t_i^0 — time of free movement along link i, f_i — volume of traffic on a link i per unit of time and p_i — capacity of link i per unit of time. The value of $d_i(f_i)$ stands for the journey time for a one vehicle on a link i depending on flow volume f_i on this link.

Nonlinear programming problem (7)–(10) can be reformulated in terms of:

$$T(f) = \min_f \sum_{i=1}^{|A|} \int_0^{f_i} t_i^0 \left(1 + \frac{u}{p_i}\right) du,$$

subject to (8)–(10).

V. Experimental results

A. Traffic assignment problem

For a numerical experiment we consider Saint-Petersburg road network. The graph G of network in presented at Fig 1. There are 109 nodes in it and 310 arcs. The nodes were chosen at the intersection of main highways and streets of the Saint-Petersburg and defined by geographical coordinates — latitude and longitude. Each arc is defined by it's BRP-function with parameters $\alpha = 0{,}15$, $\beta = 4$ as they are set in most recent practical studies on traffic assignment problem [23]. Time of free movement was calculated as the ratio of the length of an arc to average speed equaled 40 kilometres per hour.

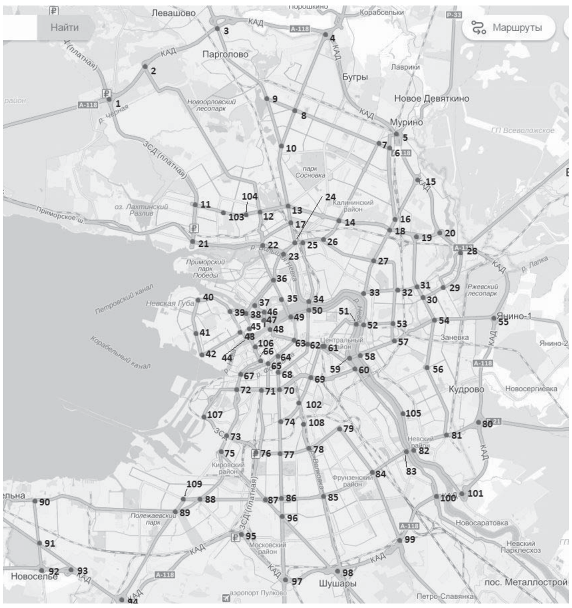

Fig. 1. Saint-Petersburg road network

Capacity of link i per unit of time p_i was calculated as:

$$p_i = k\mu p_i^0, \tag{11}$$

where p_i^0 — capacity of one traffic lane per unit of time [24]. Coefficient k in formula (11) indicates how many traffic lanes the arc has: k equals 1 if there is one lane, 1.9 — in case of two lanes, 2.7 — in case of three lanes, 3.5 — in case of four or more. Coefficient $\mu \in [0, 1]$ in formula (11) shows the dependance of the capacity of an arc on the traffic-light conditions and is defined as follows:

$$\mu = \frac{l}{l + \frac{v^2}{2}\left(\frac{1}{a} + \frac{1}{b} + v\frac{T_{all} - T_{green}}{2}\right)},$$

where l — arc length, m; a — acceleration during speed increase (1,0 m/s^2), b — deceleration during braking (1,5 m/s^2); T_{all} — the duration of the fixed cycle of traffic light, s; t_{green} — the duration of green phase, s. For highways μ equals 1 and opposite – it is about 0.5 for streets with heavy traffic. The capacity of one traffic lane per unit of time p_i^0 is defined by (12).

$$p_i^0 = \frac{3600v}{(v + 0{,}7 + 0{,}13v^2)}, \tag{12}$$

where v — the average speed. Formula (12) considers several parameters such as the safety distance between vehicles, the average length of a vehicle, the average braking distance, coefficient of adhesion, longitudinal gradient and others.

We divide time horizon into periods $Z_h = [z_{h-1}; z_h]$, $h = 2, \ldots, H$. Total link flow F^{rs} for each origin-destination pair (r, s) was defined for every period Z_k by experts of the Center of Intellectual logistics of St. Petersburg State University in collaboration with specialists of Solomenko Institute of Transport Problems of the Russian Academy of Sciences.

The user equilibrium flow distribution was found by using the Frank-Wolfe algorithm [25]. In conjunction with Dijkstra's algorithm [26] of finding shortest path we calculate the set of travel time matrices C^h corresponding to each of the periods of planning horizon:

1) 8.00 – 10.00, matrix C^1;
2) 10.00 – 12.00, matrix C^2;
3) 12.00 – 14.00, matrix C^3;
4) 14.00 – 16.00, matrix C^4;
5) 16.00 – 18.00, matrix C^5.
6) 18.00 – 20.00, matrix C^6;
7) 20.00 – till the end of routing, matrix C^7.

For further solving routing problems we define continuous piecewise function (6) for every arc on the graph G of Saint-Petersburg's road network and form resulting matrix C_{df}. We also introduce matrix C_{ff} in which values correspond to travel time on the free-flow road network.

B. Vehicle routing problem

In this subsection we will demonstrate the effectiveness of taking into account traffic load information for planning delivery routes.

For a numerical experiment we generated the set of TD-MDVRP problem instances of various dimensions with 10 to 109 clients in each example randomly chosen from Saint-Petersburg road network. We chose two depots which were in the 2-nd and 90-th nodes of the graph at Fig. 1. Service time for each client was taken to be 10 minutes. We assume the service time also includes parking time. Time limit for each route did not vary during the working day and was equal to 480 minutes (8 hours). Problem instances were solved by genetic heuristics. The algorithm is described in details in [27]. Number of generation for constructing one solution equaled 20 and there were 100 individuals in population. The results of routing are presented in Table I.

TABLE I. TD-MDVRP SOLUTIONS BY GENETIC HEURISTICS

Num. of customers	Number of cars			Total time, min		
	Depot 1	Depot 2	Total	Depot 1	Depot 2	Total
20	1	3	4	34,14	391,31	425,45
30	1	4	5	34,14	493,69	527,83
40	2	4	6	170,84	502,12	672,96
50	2	6	8	169,11	698,24	867,35
60	6	6	12	533,87	737,26	1271,13
70	7	8	15	713,33	748,51	1461,84
80	4	8	12	385,6	919,48	1305,08
90	5	9	14	497,92	958,71	1456,63
100	8	9	17	874,69	1092,4	1967,09

Numerical experiments were also held on travel time matrix with constant values. In this case the values in this matrix stand for the travel time on the free-flow road network. To perform comparison between results of MDVRP and TD-MDVRP formulations we assume notation as follows: let s^* be the solution of a problem instance on the free-flow network, and s — on the delay network. We introduce two functions $len_{ff}(s)$ and $len_{df}(s)$ which define the travel time on a route s according to matrices C_{ff} and C_{df} respectively. Table 2 contains the comparison of MDVRP and TD-MDVRP solutions on the same set of problem instances.

TABLE II. COMPARISON OF TD-MDVRP AND MDVRP SOLUTIONS

Num. of customers	TD-MDVRP solution, min.	MDVRP solution, min		Improvement percent., %.	Avg. delay of car, min.
		Expected	Real		
20	425,45	292,14	565,03	24,7	68,22
30	527,83	432,44	843,71	37,44	82,25
40	672,96	480,33	807,79	16,69	54,58
50	867,35	660,8	1241,51	30,14	72,59
60	1271,13	953,99	1615,52	21,32	55,13
70	1461,84	1089,92	1806,12	19,06	47,75
80	1305,08	986,07	1794,55	27,28	67,37
90	1456,63	1140,33	1730,71	15,84	42,17
100	1967,09	1372,19	2130,24	7,66	44,59

First column of Table 2 corresponds to the size of problem instance. In the second column the overall travel time of TD-MDVRP-solution is presented. These values can be also found in Table 1. Column of MDVRP-solution consists of two sub-columns: expected and real time. The notation of expected time means the time $len_{ff}(s^*)$, which vehicles are going to spend according to the route of the MDVRP-solution. As the formulation of MDVRP doesn't consider traffic information while generating routes, real time on road will differ from planned one. For estimation of real time on constructed route, we should combine the route s^* and travel time matrix C_{df} in case of delay-flow network. Therefore, the sub-column signed as real time on road according to the MDVPR-solution in the Table 2 is formed as follows $len_{df}(s^*)$.

As a result, travel time in TD-MDVRP solution needed for serving all customers is less than this value in case of MDVRP formulation. In the Table 2 one can also see the percentage of improvement calculated as the ratio of difference between time travel of real MDVRP- and TD-MDVRP- solution to travel time of MDVRP-route on delay flow network. Last column of Table 2 represents the average delay of the route of one car, i.e on how much minutes (in average) the travel time of one vehicle increases on delay network in comparison with planned time.

Let us consider the VRP-problem instance with the depot in the 90-th node on a graph at Fig. 1 and five customers at nodes 60, 62, 77, 85, 102. The service time of one client equaled ten minutes. Generated by genetic heuristics the VRP-solution lasts 253 minutes and TD-VRP — 232 minutes. Routes corresponding to two solutions are showed at Fig. 2 and Fig. 3.

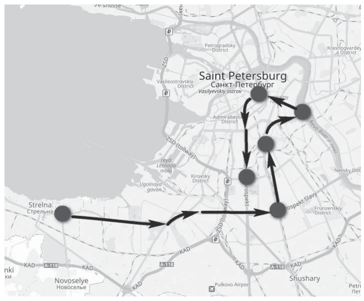

Fig. 2. VRP-solution in the test instance with the depot and five customers

TD-VRP does not include well-predictable traffic congestions and chooses to route through Saint-Petersburg Ring Road.

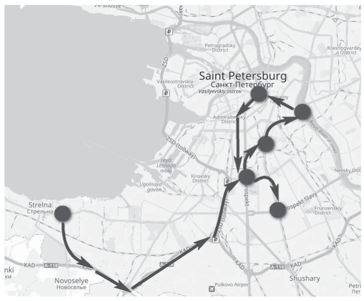

Fig. 3. TD-VRP-solution in the test instance with the depot and five customers

In some cases, one of the aims of freight forwarding companies is the reducing the overall time on road of vehicle or the reducing the fuel consumption. For this it is useful to choose beforehand the period of a day to start the routing at. The comparison of travel time of constructed routes in cases of various departure time is presented in Table 3.

One can note, there is one strongly marked rush hour — at 18:00. In the morning and afternoon (from 8:00 to 14:00) travel time differs a bit from each other. Most suitable time for staring routing is the 16:00, it will save about 100 minutes and some fuel respectively. It's worth mentioning that travel time on route depends on topology of nodes (addresses of clients in problem instance) on city graph. In this way, the most useful departure time for various problem instances can differ. This

TABLE III. TD-MDVRP ROUTING IN CASE OF DIFFERENT DEPARTURE TIMES

Num. of cust.	Departure time						
	8:00	10:00	12:00	14:00	16:00	18:00	20:00
	TD-MDVRP solution, min.						
20	425	456	460	469	440	483	390
30	527	543	541	552	542	536	503
40	672	714	686	724	675	742	693
50	867	938	909	971	874	1001	855
60	1271	1254	1408	1417	1468	1698	1416
70	1461	1519	1449	1564	1503	1619	1545
80	1305	1308	1413	1525	1477	1427	1316
90	1456	1737	1676	2007	1764	1928	1711
100	1967	2037	2113	2239	2349	2139	2093

value should be chosen individually, for example if car visits the same set of customers daily or weekly.

VI. CONCLUSION

The purpose of this paper was to give an introduction to solve the routing problems in a real network after assignment traffic flows. Time-dependent model was chosen in order to perform a realistic optimization in urban contexts. Experiments were performed on a road network of Saint-Petersburg to evaluate the model in a static and a dynamic environment. As a result, taking into account current traffic flow information leads to considerably better results in solving routing problems. The percent of improvement is up to 37% for some problem instances.

The main obstacle in applying such methods is a low accuracy of traffic information. The traffic assignment in this study took into account only the length of the roads and the number of lanes on them. Other parameters were set to be equal. For better results it would be great to analyze more factors such as traffic-light conditions for each road and intersection, changing velocity, and some mechanics factors by calculating lane capacity. Moreover, transport flows were defined by experts and can not represent an urban congested environment accurately. Processing of obtained data should be performed with the help of specialized software to model traffic flow assignment on road networks. Systems of traffic monitoring which nowadays are being widely implemented can provide data to construct origin-destination matrices for nodes of a road network.

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