Inertial Lever in the Structures of Vibrational Interactions: Estimation of Possibilities and the Peculiarities of External Excitations

Alexey I. Orlenko[1], Andrey V. Eliseev[2], Sergey V. Eliseev[3]
[1]Krasnoyarsk Institute of Railway Transport – branch of Irkutsk State Transport University, Krasnoyarsk, Russia
[2,3]Irkutsk State Transport University, Irkutsk, Russia
[1]orlenko_ai@krsk.irgups.ru, [2]eavsh@ya. [3]eliseev_s@inbox.ru

***Abstract*—A method is proposed for constructing mathematical models of mechanical oscillatory systems having an object and intermediate mass inertial elements of the lever type. The technologies of structural mathematical modeling are used, within the framework of which the mechanical oscillatory system is compared with the structural scheme of the dynamically equivalent automatic control system. Mathematical models of systems with lever inertial couplings are oriented to the estimation of the possibilities of realizing the modes of dynamic damping of oscillations with various combinations of the system parameters. The features of accounting for external influences of various types are shown. The analysis is based on the use of transfer functions of systems. The concepts of the quasi-springs and dynamic rigidity are introduced. The work is of interest to specialists in the field of machine dynamics, vibration engineering and vibration technology.**

I. Introduction

Dynamic interactions of elements of dynamic vibrational systems and vibration-proof systems, in particular, were considered in sufficient detail in the work on the theory of chains [1 ÷ 4].

Many questions related to the problems of the dynamics of oscillatory systems, which include lever mechanisms and devices for transforming motion, are reflected in recent publications [5 ÷ 8]. At the same time, in solving the problems of the dynamics of systems with lever devices, there are certain difficulties. The latter is due to the insufficient development of detailed ideas about the formation of lever interactions and the forms of their manifestation that arise in the compounds of typical elements of vibrational structures.

The proposed article deals with the formation of mathematical models of mechanical oscillatory systems with linkages formed by additional mass-inertial elements in the form of solids (or levers).

II. Some general provisions

The design scheme of a vibration protection system (Fig. 1) consisting of a mass protection object with support surfaces *I*, *II*, *III*, each of which can be the source of the corresponding kinematic harmonic disturbance (z_1, z_2, z_3). The coordinate system *y*, φ is connected with a fixed basis. The mechanical oscillatory system has two degrees of freedom, because, in addition to the object of protection (*m*), an intermediate solid body having a fulcrum (p. *O*). The solid is represented by a rigid rod (A_1, A_2); its position is determined by the angular coordinate φ. At the ends of the rods (of length l_1 and l_2) there are material points A_1, A_2 with masses m_1 and m_2. In this way an intermediate solid body has a moment of inertia.

$$J = m_1 l_1^2 + m_2 l_2^2 . \qquad (1)$$

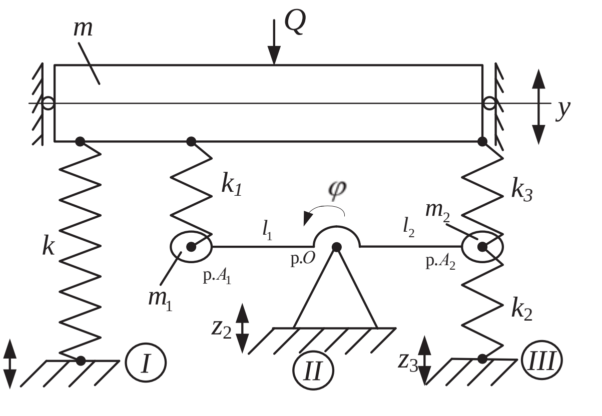


Fig. 1. The design scheme of a vibration protection system with an intermediate solid body (inertial lever of the second kind)

The system makes small fluctuations with respect to the position of static equilibrium in the coordinate system associated with the fixed basis; the center of gravity of the lever coincides with the reference point (p.*O*).

III. Construction of mathematical model: technology and system properties

The case of force perturbation, that is, $Q \neq 0$ at $z_1 = 0$, $z_2 = 0$, $z_3 = 0$. We write the expressions for the kinetic and potential energies:

$$\mathrm{T} = \frac{1}{2} m \dot{y}^2 + \frac{1}{2} (m_1 l_1^2 + m_2 l_2^2) \dot{\varphi}^2 , \qquad (2)$$

$$\Pi = \frac{1}{2}ky^2 + \frac{1}{2}k_1(y - l_1\varphi)^2 + \\ + \frac{1}{2}k_2(l_2\varphi)^2 + \frac{1}{2}k_3(y + l_2\varphi)^2. \qquad (3)$$

After a number of transformations [4], the equations of motion take the form:

$$\bar{y}(mp^2 + k + k_1 + k_3) - \bar{\varphi}(k_1l_1 - k_3l_2) = \bar{Q}, \qquad (4)$$

$$\bar{\varphi}[(m_1l_1^2 + m_2l_2^2)p^2 + k_1l_1^2 + k_2l_2^2 + k_3l_2^2] - \\ - \bar{y}(k_1l_1 - k_3l_2) = 0. \qquad (5)$$

The structural scheme of the initial system is shown in Fig. 2.

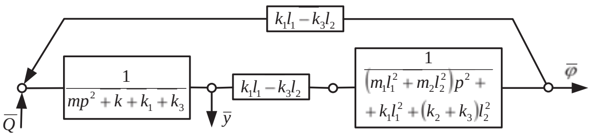


Fig. 2. Structural diagram of the vibration protection system with two degrees of freedom under force perturbation

As follows, from Figure 2 the structural diagram consists of two partial systems. Inter-partial connections in the system are among the elastic ones and are realized by a standard link with reduced rigidity

$$k' = k_1l_1 - k_3l_2. \qquad (6)$$

The input power disturbance $\bar{Q}$ in this case acts only on one input. The structural scheme in Fig. 2 can be transformed so as to exclude the coordinate $\bar{\varphi}$ (Fig. 3). We note that in the equations of motion (4), (5) and the structural schemes in Fig. 2, as well as in the following expressions and figures, the following notation is used $p = j\omega$ – the complex variable, the symbol (–) over the variable corresponds to its Laplace image [4].

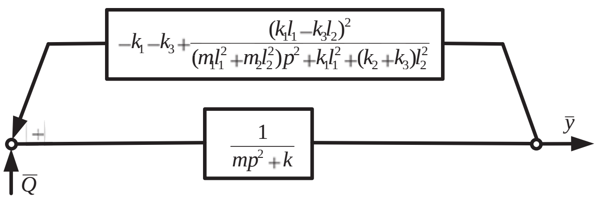


Fig. 3. Transformed structural scheme with the exception of the coordinate $\bar{\varphi}$

The basic model consists of two elements: an object of protection of mass *m* and an elastic element with rigidity *k*, resting on the support surface *I*. In Fig. 3, the basic model is interpreted by a link with a transfer function

$$W_{\text{rev}}(p) = \frac{1}{mp^2 + k}. \qquad (7)$$

The structural scheme (Fig. 3) shows the possibility of isolating a separate feedback loop relative to the base model with the transfer function (7), which has a transfer function:

$$W(p) = k_{\text{str}}(p) = \\ = \frac{\left[\begin{array}{l}(k_1 + k_3)\cdot(m_1l_1^2 + m_2l_2^2)p^2 + \\ + k_1k_3(l_1 + l_2)^2 + k_2l_2^2(k_1 + k_3)\end{array}\right]}{(m_1l_1^2 + m_2l_2^2)p^2 + k_1l_1^2 + k_2l_2^2 + k_3l_2^2} \qquad (8)$$

Expression (8), in the physical sense, determines the properties of a quasi-elastic element or the so-called quasi-spring [9]. From (8), the reduced dynamic rigidity of a quasi-spring can be found. Note that the dynamic stiffness of $W(p) = k_{\text{str}}(p)$ depends on the frequency of the external action.

IV. THE INFLUENCE OF THE STIFFNESS VALUES OF ELASTIC ELEMENTS OF QUASI-SPRING

The introduction of an intermediate solid in this problem with an object of protection of mass *m* and a force perturbation *Q* to the possibility of considering a completely defined structure (or structural formation) in the form of an inertial lever. The situation can be represented as the action of a quasi-spring that determines the functional capabilities of a mass-inertial lever of the second kind with a reference point *O* (Fig. 1). We consider a number of special cases.

If $k_3 = 0$, then

$$W'(p) = \frac{k_1[(m_1 + m_2i^2)p^2 + k_2i^2]}{(m_1 + m_2i^2)p^2 + k_1 + k_2i^2}, \qquad (9)$$

where $i = \frac{l_2}{l_1}$ – gear ratio of the lever of the second kind.

If $k_2 = 0,\ k_3 = 0$, then

$$W''(p) = \frac{k_1(m_1 + m_2i^2)p^2}{(m_1 + m_2i^2)p^2 + k_1}. \qquad (10)$$

If $k_1 \to \infty$, then it follows from (10) that

$$W'''(p) = (m_1 + m_2i^2)p^2. \qquad (11)$$

The resulting expression (11) corresponds, in the physical sense, to attaching additional masses to the object of protection $(m_1 + m_2i^2)$.

The transfer function of the system as a whole under the force perturbation of the $\bar{Q}$ has the form

$$W(p) = \frac{\bar{y}}{\bar{Q}} = \\ = \frac{(m_1l_1^2 + m_2l_2^2)p^2 + k_1l_1^2 + (k_2 + k_3)l_2^2}{\left[\begin{array}{l}(mp^2 + k + k_1 + k_3)\cdot[(m_1l_1^2 + m_2l_2^2)p^2 + \\ + k_1l_1^2 + (k_2 + k_3)l_2^2] - (k_1l_1 - k_3l_2)^2\end{array}\right]}. \qquad (12)$$

In the system, a mode of dynamic damping of oscillations at a frequency:

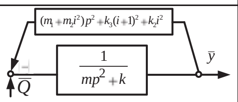
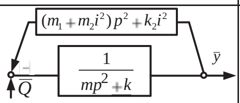
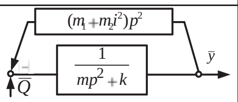
$$\omega_{\text{dyn}}^2 = \frac{k_1 l_1^2 + (k_2 + k_3) l_2^2}{m_1 l_1^2 + m_2 l_2^2} = \frac{k_1 + (k_2 + k_3) i^2}{m_1 + m_2 i^2}, \quad (13)$$

If $k_1 \to \infty$, then the system loses one degree of freedom:

$$W^{\text{IV}}(p) = \frac{\bar{y}}{\bar{Q}} = \\ = \frac{1}{(m + m_1 + m_2 i^2) p^2 + (k_2 + k_3) i^2 + k}. \quad (14)$$

In the Table I lists, for comparison, the types of structural schemes of the system and the features of their transfer functions.

TABLE I. VARIETIES OF STRUCTURAL DIAGRAMS OF THE SYSTEM AND FEATURES OF THEIR TRANSFER FUNCTIONS

Parameters	Schemes	Transmission function
$k_1 \to \infty$		$W'(p) = \frac{\bar{y}}{\bar{Q}} = \frac{1}{\left[(m + m_1 + m_2 i^2) p^2 + + (k_2 + k_3) i^2 + k \right]}.$
$k_1 \to \infty$ and $k_3 = 0$		$W'(p) = \frac{\bar{y}}{\bar{Q}} = = \frac{1}{\left[(m + m_1 + m_2 i^2) p^2 + + k_2 i^2 + k \right]}.$
$k_1 \to \infty$, $k_3 = 0$ and $k_2 = 0$		$W'(p) = \frac{\bar{y}}{\bar{Q}} = = \frac{1}{\left[(m + m_1 + m_2 i^2) \times \times p^2 + k \right]}.$

With force perturbation, the inertial lever, which is part of the quasi-spring, not only forms a complex elastic element with dynamic rigidity, depending on the frequency of the external action, but also provides the possibility of the emergence of specific modes: Dynamic vibration damping mode and motion mode with zero dynamic stiffness. The latter mode represents the motion of the elements of the system in a form corresponding to the shape of the vibration with free motions of the original system.

V. THE CONSTRUCTION OF A MATHEMATICAL MODEL UNDER KINEMATIC PERTURBATION TAKES INTO ACCOUNT THE MOTIONS OF THREE SUPPORT SURFACES (FIG. 1)

Developing a more general approach for $z_2 \neq 0$, taking into account the motion of the support surface and the rotation angle φ, we obtain expressions for the velocities of points A_1 and A_2 in absolute motion

$$v_{A_1} = -l_1 \dot{\varphi} + \dot{z}_2, \; v_{A_2} = l_2 \dot{\varphi} + \dot{z}_2. \quad (15)$$

In this case, the expression for the kinetic energy takes the form:

$$\text{T} = \\ = \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m_1 (-l_1 \dot{\varphi} + \dot{z}_2)^2 + \frac{1}{2} m_2 (l_2 \dot{\varphi} + \dot{z}_2)^2, \quad (16)$$

from which it follows that

$$\frac{1}{2} J \dot{\varphi}^2 = \\ = \frac{1}{2} m_1 (-l_1 \dot{\varphi} + \dot{z}_2)^2 + \frac{1}{2} m_2 (l_2 \dot{\varphi} + \dot{z}_2)^2 = \\ = \frac{1}{2} (m_1 l_1^2 + m_2 l_2^2) \dot{\varphi}^2 + \dot{\varphi} (m_2 l_2 - m_1 l_1) \dot{z}_2 + \\ + \frac{1}{2} \dot{z}_2^2 (m_1 + m_2). \quad (17)$$

Since the center of gravity of the intermediate solid body coincides with the center of its rotation, then $m_2 l_2 - m_1 l_1 = 0$. The expression for the potential energy with allowance for the kinematic perturbations can be written:

$$\Pi = \frac{1}{2} k (y - z_1)^2 + \frac{1}{2} k_1 (y - l_1 \varphi - z_2)^2 + \\ + \frac{1}{2} k_2 (l_2 \varphi - z_2 - z_3)^2 + \frac{1}{2} k_3 (y + l_2 \varphi - z_2)^2. \quad (18)$$

Formation of expression (18) takes into account the features of the motion of an intermediate solid body (or inertial lever of the second kind)

The coefficients of the equations of motion are given in Table II.

TABLE II. COEFFICIENTS OF THE EQUATION OF MOTION OF THE SYSTEM IN COORDINATES $\bar{y}$, $\bar{\varphi}$

a_{11}	**a_{12}**
$mp^2 + k + k_1 + k_3$	$-k_1 l_1 + k_3 l_2$
a_{21}	**a_{22}**
$-k_1 l_1 + k_3 l_2$	$Jp^2 + k_1 l_1^2 + k_2 l_2^2 + k_3 l_2^2$
Q_1	**Q_2**
$kz_1 + k_1 z_2 + k_3 z_2$	$-k_1 l_1 z_2 + k_3 l_2 z_2 + k_2 l_2 z_3 + k_2 l_2 z_2$

Let us consider the case of a support on a common support surface, which determines the input perturbation for the same movements of the support surface *I* and *II*:

$$\overline{Q}_y = \bar{z}(k + k_1 + k_3) \quad (19),$$

$$\overline{Q}_\varphi = \bar{z}(-k_1 l_1 + k_2 l_2 + k_3 l_2). \quad (20)$$

In this case it is assumed that $\bar{z}_1 = \bar{z}_2 = \bar{z}$, $\bar{z}_3 = 0$. We find the transfer function of the system with allowance for the effects of the input disturbances $\overline{Q}_y$ and $\overline{Q}_\varphi$:

$$W_0'(p) = \frac{\bar{y}}{\bar{z}} = \frac{\left[\begin{array}{l}(k + k_1 + k_3)\cdot(Jp^2 + k_1 l_1^2 + k_2 l_2^2 + k_3 l_2^2) + \\ + (-k_1 l_1 + k_2 l_2 + k_3 l_2)\cdot(k_1 l_1 - k_3 l_2)\end{array}\right]}{A_0}, \quad (21)$$

where

$$A_0 = (mp^2 + k + k_1 + k_3) \times \times (Jp^2 + k_1 l_1^2 + k_2 l_2^2 + k_3 l_2^2) - -(k_1 l_1 - k_3 l_2)^2. \quad (22)$$

We group the numerator and denominator (21) with the selection of k_1, omitting the other terms of the expressions, assuming that $k_1 \to \infty$, we obtain

$$W_0'(p) = \frac{\bar{y}}{\bar{z}} = \frac{Jp^2 + k_2 l_2^2 + k_3 l_2^2 + l_1^2(k + k_3) + k_2 l_1 l_2 + 2k_3 l_1 l_2}{(ml_1^2 + J)p^2 + k_2 l_2^2 + k_3 l_2^2 + l_1^2(k + k_3) + 2k_3 l_1 l_2}. \quad (23)$$

We divide the numerator and denominator (23) by l_1^2, then

$$W_0''(p) = \frac{\bar{y}}{\bar{z}} = \frac{\left[\begin{array}{l}(m_1 + m_2 i^2)p^2 + (k_2 + k_3)i^2 + \\ + k + k_3 + i(k_2 + 2k_3)\end{array}\right]}{\left[\begin{array}{l}(m + m_1 + m_2 i^2)p^2 + \\ + (k_2 + k_3)i^2 + k + k_3 + 2k_3 i\end{array}\right]}. \quad (24)$$

Table III provides comparative data for a different combination of parameters.

VI. Elastic fixation of an intermediate solid body at the fulcrum

Consider the design scheme of the system (Fig. 4) where the intermediate rigid body has an elastic element at the point O with rigidity k_0.

TABLE III. Variants of transfer functions for various values of the rigidity of elastic elements

Parameters	Transmission function
$k_1 \to \infty$	$W_2(p) = \frac{\bar{y}}{\bar{z}} = \frac{\left[\begin{array}{l}Jp^2 + k_2 l_2^2 + k_3 l_2^2 + \\ + l_1^2(k + k_3) + k_2 l_1 l_2 + 2k_3 l_1 l_2\end{array}\right]}{\left[\begin{array}{l}(ml_1^2 + J)p^2 + k_2 l_2^2 + \\ + k_3 l_2^2 + l_1^2(k + k_3) + 2k_3 l_1 l_2\end{array}\right]}$
$k_1 \to \infty$ and $k_3 = 0$	$W_3(p) = \frac{\bar{y}}{\bar{z}} = \frac{(m_1 + m_2 i^2)p^2 + k_2 i^2 + k + ik_2}{(m + m_1 + m_2 i^2)p^2 + k_2 i^2 + k}$
$k_1 \to \infty$, $k_3 = 0$ and $k_2 = 0$	$W_3(p) = \frac{\bar{y}}{\bar{z}} = \frac{(m_1 + m_2 i^2)p^2 + k}{(m + m_1 + m_2 i^2)p^2 + k}$

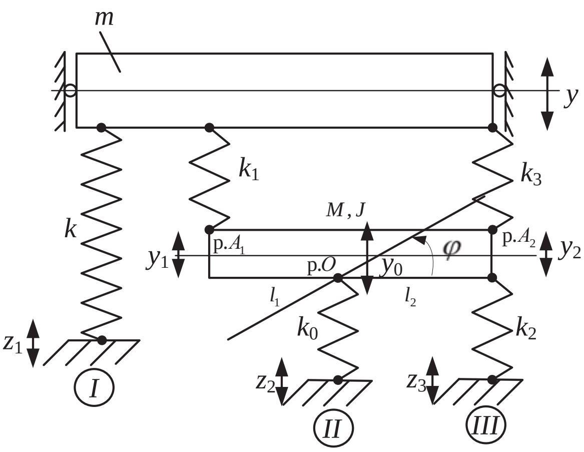


Fig. 4. Calculation scheme of a vibration protection system with an intermediate solid on an elastic support

We write the expressions for the kinetic and potential energies

$$\mathrm{T} = \frac{1}{2}m\dot{y}^2 + \frac{1}{2}M\dot{y}_0^2 + \frac{1}{2}J\dot{\varphi}^2, \quad (25)$$

$$\Pi = \frac{1}{2}k(y - z_1)^2 + \frac{1}{2}k_1(y - y_1)^2 + + \frac{1}{2}k_2(y_2 - z_3)^2 + \frac{1}{2}k_3(y - y_2)^2 + + \frac{1}{2}k_0(y_0 - z_2)^2. \quad (26)$$

Relations between the coordinates of the system

$$\begin{cases} y_0 = ay_1 + by_2, \varphi = c\cdot(y_2 - y_1), \\ y_1 = y_0 - l_1\varphi, y_2 = y_0 + l_2\varphi, \\ a = \frac{l_2}{l_1+l_2}, b = \frac{l_1}{l_1+l_2}, c = \frac{1}{l_1+l_2}. \end{cases} \quad (27)$$

Using the above techniques, we can obtain a system of equations of motion in the coordinates $\bar{y}$, $\bar{\varphi}$ and $\bar{y}_0$. The coefficients of the equations are given in Table IV.

TABLE IV. COEFFICIENTS OF THE EQUATION OF MOTION OF THE SYSTEM IN COORDINATES $\bar{y}$, $\bar{\varphi}$ AND $\bar{y}_0$

a_{11}	a_{12}	a_{13}
$mp^2 + k + k_1 + k_3$	$k_1l_1 - k_3l_2$	$-k_1 - k_3$
a_{21}	a_{22}	a_{23}
$k_1l_1 - k_3l_2$	$Jp^2 + k_1l_1^2 + k_2l_2^2 + k_3l_2^2$	$-k_1l_1 + k_2l_2 + k_3l_2$
a_{31}	a_{32}	a_{33}
$-k_1 - k_3$	$-k_1l_1 + k_2l_2 + k_3l_2$	$Mp^2 + k_0 + k_1 + k_2 + k_3$
Q_y	Q_φ	Q_{y0}
kz_1	$k_2l_2z_3$	$k_2z_3 + k_0z_2$

We use the Cramer formulas [3] to determine the motion along the coordinate $\bar{y}$:

$$\bar{y} = \frac{\left[\begin{array}{l}\bar{Q}_y(a_{22}a_{33} - a_{23}^2) + \bar{Q}_\varphi(a_{13}a_{32} - a_{12}a_{33}) + \\ + \bar{Q}_{y_0}(a_{12}a_{23} - a_{13}a_{22})\end{array}\right]}{\left[\begin{array}{l}a_{11}a_{22}a_{33} - a_{11}a_{23}^2 - a_{22}a_{13}^2 - a_{33}a_{12}^2 + \\ + 2a_{12}a_{23}a_{31}\end{array}\right]}. \quad (28)$$

We rewrite (28) in the form, assuming that $\bar{z}_1 = \bar{z}_2 = \bar{z}$, $\bar{z}_3 = 0$:

$$W_0^{III}(p) = \frac{\bar{y}}{\bar{z}} = \\ = \frac{k(a_{22}a_{33} - a_{23}^2) + k_0(a_{12}a_{23} - a_{13}a_{22})}{A_0'}, \quad (29)$$

where

$$A_0' = a_{11}a_{22}a_{33} - a_{11}a_{23}^2 - a_{22}a_{13}^2 - a_{33}a_{12}^2 + \\ + 2a_{12}a_{23}a_{31}. \quad (30)$$

The values of a_{ij} are given in Table IV.

For $k_0 \to \infty$, we obtain that

$$W_0^{IV}(p) = \frac{\bar{y}}{\bar{z}} = \\ = \frac{\left[\begin{array}{l}(k + k_1 + k_3)\cdot(Jp^2 + k_1l_1^2 + k_2l_2^2 + k_3l_2^2) + \\ + (k_1l_1 - k_3l_2)\cdot(-k_1l_1 + k_2l_2 + k_3l_2)\end{array}\right]}{\left[\begin{array}{l}(mp^2 + k + k_1 + k_3)\times \\ \times(Jp^2 + k_1l_1^2 + k_2l_2^2 + k_3l_2^2) - (k_1l_1 - k_3l_2)^2\end{array}\right]} \quad (31)$$

Formula (31) coincides with (21), which makes it possible to verify the validity of the method for constructing mathematical models of mechanical vibrational systems with solids, which form joints [3], [10].

VII. CONCLUSION

Leverage links in mechanical oscillatory systems manifest themselves in various forms, which is not always associated with the usual notions of lever mechanisms. Leverage properties are also provided by intermediate solids introduced into the composition of vibrational systems including, and vibration-proof, for the implementation of special dynamic modes of operation and control of the dynamic state of technical objects. It is shown that the leverage links that arise during the dynamic interactions of system elements can be considered using more detailed representations. Such mathematical models can be obtained on the basis of the proposed techniques and interpretation of results by transition to the use of transfer functions.

The definition of leverage links, in terms of their type evaluation, and dynamic properties, can be realized by introducing intermediate solid bodies for which the joints of the links are applied at certain points of the solid body where the support of the lever.

It is shown that an intermediate solid body can essentially be regarded as an inertial lever. With the "zeroing" of the mass inertial properties of the lever, the number of degrees of freedom of the system. In particular, the mandatory reduction in the number of degrees per unit occurs when the anchor point "fixed".

In general, the lever, in comparison with such typical elastic and dissipative elements, is a more complex link. The transfer function of such a link in structural approaches can be represented by a fractional-rational expression of the second order.

The linkage plays an essential role in the formation of the dynamic properties of the system, forming a spatial system of elements and conditions for transforming the movements of elements that create different modes of dynamic states.

A method and technology for constructing mathematical models is proposed, which makes it possible to implement approaches related to the search for and evaluation of rational constructive and technical solutions in developing methods and means of vibration protection, in particular*

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