Analysis and Simulation of NAMA Algorithm for Decentralized Network

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Abstract

The object of research is modern decentralized communication system, which can contain quite a large number of users. Actual problem in this area is the study of algorithms of dynamic scheduling of users access to the communication channel. In this work algorithm NAMA is considered and the results of the analysis of its main characteristics are given.

Index Terms: Multiple access, Decentralized communication system, Dynamic time division access.

I. INTRODUCTION

Nowadays there are a lot of decentralized data communication systems that use time division multiple access. These systems may contain a large number of users, so a static time division access does not provide the required quality of service [1]. This paper presents an algorithm in which the resources are reserved by dynamic scheduling.

II. MAIN PART

Following [2], the system can be described with a following model. Suppose that the system contains V users. It is assumed that the user can directly communicate with users which are at a distance no greater than r from him and indirectly – at a distance no greater than $2r$ from him.

Model of decentralized system with a dynamic schedule can be represented as a graph $G=(V,E)$, where V is set of users, E is set channels between subscribers. If two users u and v located at a distance not more than r from each other, there is a channel $(u,v)\in E$. In this case users u and v can communicate with each other. We call them one-hop neighbors. Denoted by K_i^1- set of one-hop neighbors of user i. We call two users who don't have a common edge, but have a common one-hop neighbor as two-hop neighbors. Thus the set $K_i^1 \cup \left(\bigcup_{j\in K_i^1} K_j^1 \right) \setminus i$ is one-hop and two-hop neighbors of user i. Example of a decentralized system model is shown in Fig. 1 (a). In this example, the neighbors of user i are selected (number 1 means one-hop neighbors, number 2 – two-hop neighbors).

The system time is divided into the slots. All users transmit messages only at the beginning of the slot. Denote "collision" as event when two or more neighbor users transmit in one slot.

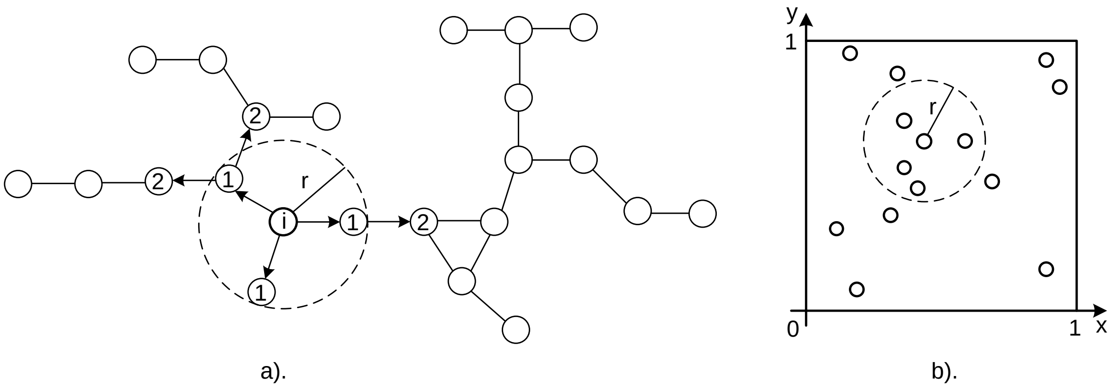

Fig. 1. Model of a decentralized system with dynamic scheduling

To simplify the model we describe the system model as follows [2]. Consider two-dimensional area of size $S = 1$ (a square with side equal 1 and the coordinates of the lower left corner equal $(0,0)$). In this system we will use a Poisson input stream, that is going to generate the number of users in the area by the Poisson law with intensity $\left(\frac{the\ number\ of\ new\ sessions}{slot \cdot area}\right)$. Let us generate two coordinates for each user as uniformly distributed random numbers. System model with a dynamic schedule is shown in Fig. 1 (b).

In papers [2, 3, 4] for the mentioned model of decentralized system time division algorithm was described, called Node Activation Multiple Access (NAMA). Using this algorithm in each slot only those users who are not neighbors will transmit. Therefore, this algorithm allows to transmit data without collision. The main indicator of system performance is the average number of transmitted packets by one user in one slot. The aim of this paper is to show the dependence of the main indicator on the radius r.

Based on paper [2] and the previously described model, we present analysis of the main indicator of system performance and algorithm NAMA. We introduce the random variable – the number of one-hop neighbors of user i, and denote it as N_1. We calculate the expectation of N_1: $n_1 = E[N_1]$. Introduce $p(k,S) = \frac{(\lambda S)^k}{k!} e^{-\lambda S}$ is the probability that there are k users in area.

The average number of users in the area of size S is λS.

Since one-hop neighbors are users at a distance no greater than r, then the average number of one-hop neighbors is $n_1 = \lambda \pi r^2$.

We introduce the random variable N_2 – the number of two-hop neighbors of user i and calculate the expectation of N_2: $n_2 = E[N_2]$. In [2] the expectation of N_2 was obtained, but the detailed explanation was omitted. We consider calculation of n_2 in details. Two-hop neighbors are users who have a common neighbor, as shown in Fig. 2. In this figure, the users are two-hop neighbors, since they have a common neighbor C. To calculate the average number of two-hop neighbors we find the number of users that have common neighbors i and j. We denote the distance between nodes i and j as $d(i,j) = l \cdot r$ where $l \in [1,2]$.

The angle α can be calculated as follows:

$$\cos\alpha(l) = \frac{\frac{lr}{2}}{r} = \frac{l}{2} \Rightarrow \alpha = \arccos\frac{l}{2}.$$

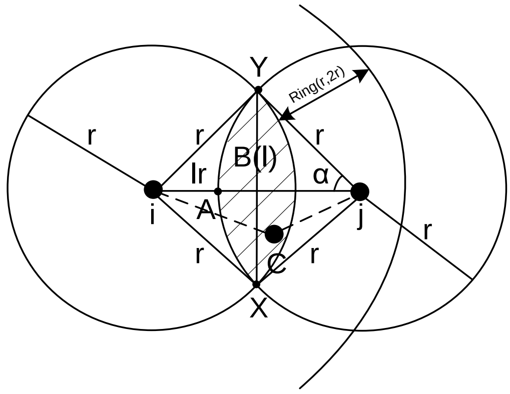

Fig. 2. Two-hop neighbors

Area of the segment XAY :

$$S_{segmentXAY} = S_{sectorXYj} - S_{triangleXYj} = \frac{r^2}{2}2\alpha - \frac{1}{2}\frac{lr}{2}2r\sin\alpha =$$

$$= r^2\left(\arccos\frac{l}{2} - \frac{l}{2}\sin\left(\arccos\frac{l}{2}\right)\right) = r^2\left(\arccos\frac{l}{2} - \frac{l}{2}\sqrt{1-\left(\frac{l}{2}\right)^2}\right)$$

Area of the hatched region can be found as the sum of the areas of the two segments XAY . Let $\alpha(l) = \arccos\frac{l}{2} - \frac{l}{2}\sqrt{1-\left(\frac{l}{2}\right)^2}$, Then area of the hatched region S' is:

$$S' = 2\cdot S_{segmentXAY} = 2r^2\alpha(l).$$

Consequently, the average number of users in the hatched area

$$B(l) = 2\lambda r^2\alpha(l),$$

$1-e^{-B(l)}$ is probability of an event that, in the shaded area will be at least one user. Then the average number of users in the interval $l\cdot r$, which have common neighbor with user i: $\int_1^2 l\cdot r\left(1-e^{-B(l)}\right)dl$, since $l\in[1,2]$. Adding up all users covered by the ring $(r,2r)$ around the user i, get the average number of two-hop neighbors:

$$n_2 = \lambda 2\pi r\int_1^2 l\cdot r\left(1-e^{-B(l)}\right)dl = \lambda\pi r^2\int_1^2 2l\left(1-e^{-B(l)}\right)dl.$$

Thus the average number of one-hop and two-hop neighbors of user i is:

$$n = n_1 + n_2 = n_1\left(1+\int_1^2 2l\left(1-e^{-B(l)}\right)dl\right).$$

In paper [2] claims that the number of competitors is Poisson distributed variable with mean n. Following this statement, the probability of an event that randomly selected station will transmit in the slot:

$$T(n) = \sum_{k=1}^{\infty} \frac{1}{k+1} \frac{n^k}{k!} e^{-n},$$

where n is average number of competitors.

For algorithm NAMA average number of competitors is the average number of neighbors, that is n. Therefore, average number of transmitted packets in the slot for algorithm NAMA:

$$q_{NAMA} = T(n).$$

This is not correct, because distribution of $N_1 + N_2$ is not Poisson. Note, that N_1 and N_2 are Poisson distributed variables, but they are dependent, therefore $N_1 + N_2$ is not Poisson distributed.

Moreover in this paper dynamic scheduling algorithm has been simulated according to paper [2]. In the algorithm NAMA in each slot the number of competitors of user A is calculated as the number of its neighbors M_A. One user from the set $M_A \cup A$ is selected according to some rule. This user will transmit. L slots were simulated to obtain an estimation of the capacity of the NAMA algorithm. And q_{NAMA} was calculated as $q_{NAMA} = \frac{L_A}{L}$, where L_A is the number of slots where user A transmitted.

In Fig. 3 shows dependence of average number transmitted packets in the slot and range obtained by numerical methods [2], and simulation usage.

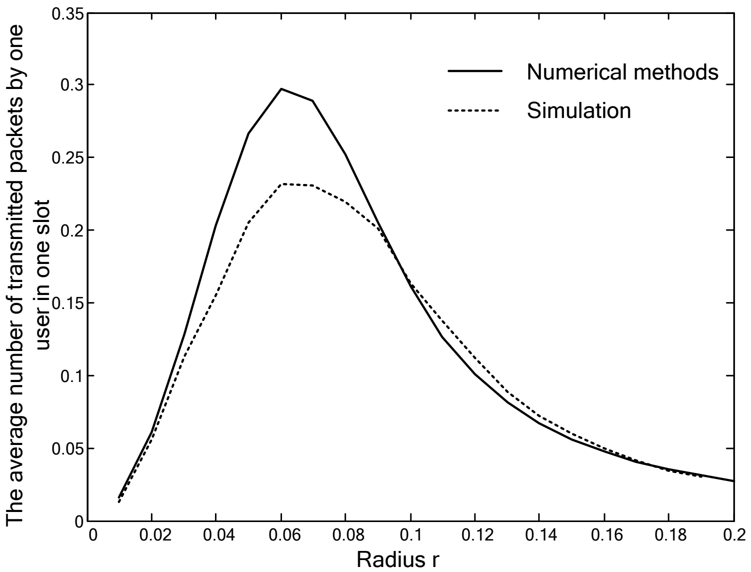

Fig. 3. The dependence of average number transmitted packets in the slot by one of the users on radius r

III. CONCLUSION

The obtained results show, that total number of neighbors is not Poisson distributed. Thus, the method described in [2] can be considered as approximation. The simulations show, that real values differ from approximation by 10%. Thus, developing of new numerical analysis methods of decentralized systems with dynamic scheduling is still actual.

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